The Numerical Solution of lntegro-partial differential equations with sixth-Degree B-Spline Functions

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Abstract. In this paper, we consider the approximate solution of the following problem

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial x^2}+\alpha\frac{\partial u}{\partial x}+\int_0^t k(t,s,u(x,s))ds = f(t, x), \qquad a \le x \le b, \quad t \in (0,T)$$

$$u\big|_{x=a}=0, \quad u\big|_{x=b}=0, \quad t\in(0,T)$$

$$u\big|_{t=0}=u_0(x), \qquad a\le x\le b.$$

To solve this problem, we introduce a new nonstandard time discretization scheme. A proof of convergence of the approximate solution is given and error estimates are derived. The numerical results obtained by the suggested technique are compared with the exact solution of the problem. The numerical solution displays the expected convergence to the exact one as the mesh size is refined, the numerical solution displays the expected convergence to the exact one as the mesh size is refined.the numerical solution displays the expected convergence to the exact one as the mesh size is refined.

Keywords. Integro-partial differential equation, Time discretization, Collocation B-spline, Numerical methods.

1. Introduction

The principal aim of this paper is to describe an approximate solution for a parabolic integro-differential equation representing heat conduction in material with positive memory. Classically, a heat conduction phenomenon is represented by a parabolic partial differential equation with an infinite heat propagation speed; this is a puzzling contradiction to the physics. Indeed, the material property of the past influences on that of the present, and therefore the heat propagation can be better understood if it is represented by an integro-differential equation rather than it is modeled by the usual parabolic equations.

It is essential to take into account the effect of past history while describing the system as a function at a given time. Consider for example, a physical situation which gives rise to a parabolic partial integro-differential equation of the form

$$\frac{\partial y}{\partial t} - \Delta y + \int_0^t k(t, r)\, g(y(r, x))\, dr = f \qquad (t, x) \in (0, T) \times \Omega, \tag{1.1}$$

$$y(0, x) = y_0(x), \qquad x \in \Omega, \tag{1.2}$$

$$y(t, x) = 0 \qquad (t, x) \in (0, T) \times \partial\Omega, \tag{1.3}$$

where $\Omega \subset R^n$ is a connected bounded domain with smooth boundary $\partial\Omega,$ is a feedback heat control in the interior of some heat conduction medium, where the control mechanism possesses some intertia or a similar control situation for a reaction-diffusion problem. In the analysis of space time dependent nuclear reactor dynamics, if the effect of a linear temperature feedback is taken into consideration and the reactor model is considered as an infinite rod, then the one group neutron flux $y(t, x)$ and the temperature $\upsilon(t, x)$ in the reactor are given by the following coupled equation (see [1]):

$$\frac{\partial y}{\partial t} - (k(x) y_x)_x = (c_1 \upsilon + c_2 - 1) \sum\nolimits_f y(x) \qquad (t > 0, -\infty < x < \infty), \tag{1.4}$$

$$\rho c \frac{\partial \upsilon}{\partial t} = c_3 \sum\nolimits_g y, \tag{1.5}$$

where a is the diffusion coefficient and $\sum_f, \sum_g, \rho, c, c_i = (i = 1, 2, 3)$ are physical quantities. By integrating the second equation in (1.4) in the interval $(0, t)$ and substituting it into the first equation, we obtain the following nonlinear integro-differential equation:

$$\frac{\partial y}{\partial t} - (k(x) y_x)_x = \beta y \int_0^t y(r, x)\, dr + by, \quad (t < 0, \quad -\infty < x < \infty), \tag{1.6}$$

where β, b are the constants associated with the initial temperature and various physical parameters. However, in the actual reactor systems, the temperature is a function of position $x,$ which may be one, two or three dimensional. Thus it is more realistic to consider the heat equation for y in a higher dimensional spatial domain (see [2-5]). Here we consider a more general system of integro-differential equations of the form:

$$\frac{\partial y}{\partial t} - L\, y = g(t, x) + f\left(t, x, y, \int_0^t K(t, x, r, y(r, x))\, dr\right) \qquad (t, x) \in (0, T) \times \Omega, \tag{1.7}$$

$$y(0, x) = y_0(x), \qquad x \in \Omega, \tag{1.8}$$

$$d(x, t) \frac{\partial y}{\partial v} + y = 0 \qquad (t, x) \in (0, T) \times \partial\Omega, \tag{1.9}$$

where Ω is a connected bounded domain in R^n with smooth boundary $\partial\Omega$ and

$$L\, y = \sum_{i, j=1}^{n} a_{ij}(x) \frac{\partial^2 y}{\partial x_i\, \partial x_j} + \sum_{i=1}^{n} a_i(x) \frac{\partial y}{\partial x_i} - a_0(x) y. \tag{1.10}$$

The existence, uniqueness and asymptotic behavior of solutions of the system of the form (1.1)-(1.3) have been studied in [3]. This problem governs many physical systems occurring in diffusion problems and includes (1.4) and (1.5) as a special case.

In this paper, we study the equations which arise in many applications (e.g., [6, 7] and the references).

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + \alpha \frac{\partial u}{\partial x} + \int_0^t k(x, s, u(t, s))\, ds = f(t, x), \qquad a \le x \le b, \quad t \in (0, T) \tag{1.11}$$

$$u\big|_{x=a} = g_1(t), \quad u\big|_{x=b} = g_2(t), \quad t \in (0, T) \tag{1.12}$$

$$u\big|_{t=0} = u_0(x), \qquad a \le x \le b, \tag{1.13}$$

where α is a constant advection velocity and k a constant diffusivity, the integral is called memory term, $k(x, s)$ is the kernel function satisfying

$$\max_{x \in I} \left| k(x, s) \right| \le C \left| A(x, s)\, K_\rho(x - s) \right|, \tag{1.14}$$

Where A is sufficiently smooth in x and s, and the Hammerstein kernel

$$K_\rho(x - s) = \begin{cases} (x - s)^{-\rho}, & 0 < \rho < 1 \\ K(x - s), & \text{otherwise,} \end{cases} \tag{1.15}$$

K is smooth function, $K_\rho(x - s) = (x - s)^{-\rho}$ is said to be weakly singular kernel.

Solution of Integro-partial differential equations has recently attracted much attention of research. The motivation for such problems lies in different branches of physics, in rtheology, and especially in the theory of parabolic type. There are several methods for solving integro-differential equations, in (1988) E. G. Yanik and G. Fairweather use finite element methods for solving integro-differential equation of parabolic type [8]. In (1989) M. N. Leroux and V. Thomèe use Numerical solution of semilinear integro-differential equations of parabolic type with non smooth data [9]. The stability of Ritz-Volterra projections and error estimates for finite element methods for a class of integro-differential equations of parabolic type is studied by Y. Lin and T. Zhang [10]. In (992), A. K. Pani, V. Thomèe, and L.B. Wahlbin use Numerical methods for hyperbolic and parabolic integro-differential equations [11]. Global and blow-up solutions of a class of semilinear integro-differential equation, by Cui Shang-bin and Ma Yu-lan in (1994) [12]. I. H. Sloan and V. Thomèe, use Time discretization of an integro-differential equation of parabolic type [13].

Our contribution in this paper is to develop a new algorithm for solving partial integro-differential equations in one dimensional space with non-homogeneous Dirichlet boundary conditions. The suggested numerical scheme starts with the discretization in time by the 2-point Euler backward finite difference method. After that we deal with a combination of the compact finite difference method and the trapezoidal rule for calculating the integral term and then we use a collocation method to compute the unknown function and finally the obtained system of algebraic equations is solved by iterative methods. The proposed technique is programmed using ***Matlab ver. 7.8.0.347 (R2009a). Toolbox based GA ver. 2.4.1.***

The paper is organized as follows: Section 2 is devoted to introducing the definition of the spline function. Section 3 is devoted to describe and analyze a time discretization scheme. The convergence of the discrete sequence of iterations is shown in Section 4. Section 5 concerns the error estimates for the approximate solution. The numerical solution of the partial integro-differential equation by using the collocation method is stated in section 6. Finally, in Section 7, the proposed scheme is directly applicable to solve some numerical example to support the efficiency of the suggested numerical scheme. . Conclusions are drawn in Section 4.

2. The Sixth-Degree B-Splines

In this section, sixth-degree B-splines are used to construct numerical solutions to the partial integro-differential problem discussed in section 4. A detailed description of B-spline functions generated by subdivision can be found in [13]. Consider equally-spaced knots of a partition $\Delta: a = x_0 < x_1 < \cdots < x_n = b$ on $[a, b]$. Let $S_6[\Delta]$ be the space of continuously-differentiable, piecewise, sixth-degree polynomials on Δ. That is, $S_6[\Delta]$ is the space of sixth-degree spline on Δ. Consider the B-spline basis in $S_6[\Delta]$. The B-splines are defined in [13] as

$$B_0(x) =$$

$$= \frac{1}{720\,h^6}\begin{cases} x^6, & 0 \le x < h, \\ -6\,x^6 + 42\,h\,x^5 - 105\,h^2\,x^4 + 140\,h^3\,x^3 - 105\,h^4\,x^2 + 42\,h^5\,x - 7\,h^6, & h \le x < 2h \\ 15\,x^6 - 210\,h\,x^5 + 1155\,h^2\,x^4 - 3220\,h^3\,x^3 + 4935\,h^4\,x^2 - 3990\,h^5\,x + 1337\,h^6, & 2h \le x < 3h, \\ -20\,x^6 + 420\,h\,x^5 - 3570\,h^2\,x^4 + 15680\,h^3\,x^3 - 37590\,h^4\,x^2 + 47040\,h^5\,x - 24178\,h^6, & 3h \le x < 4h, \\ 15\,x^6 - 420\,h\,x^5 + 4830\,h^2\,x^4 - 29120\,h^3\,x^3 + 96810\,h^4\,x^2 - 168000\,h^5\,x + 119182\,h^6, & 4h \le x < 5h, \\ -6\,x^6 + 210\,h\,x^5 - 3045\,h^2\,x^4 + 23380\,h^3\,x^3 - 100065\,h^4\,x^2 + 225750\,h^5\,x - 208943\,h^6, & 5h \le x < 6h, \\ x^6 - 42\,h\,x^5 + 735\,h^2\,x^4 - 6860\,h^3\,x^3 + 36015\,h^4\,x^2 - 100842\,h^5\,x + 117649\,h^6, & 6h \le x < 7h, \end{cases}$$

where

$$B_i(x) = B_0(x - ih), \qquad i = -3, -2, \cdots \tag{2.1}$$

To solve integro-partial differential equations, the B_i and their derivatives, evaluated at the nodal points, are needed. Their coefficients are given in Table 1.

Table 1. Coefficients of B_i and its derivatives.

	x_i	x_{i+1}	x_{i+2}	x_{i+3}	x_{i+4}	x_{i+5}	x_{i+6}	x_{i+7}
B_i	0	1	57	302	302	57	1	0
B_i'	0	$\frac{6}{h}$	$\frac{150}{h}$	$\frac{240}{h}$	$\frac{-240}{h}$	$\frac{-150}{h}$	$\frac{-6}{h}$	0
B_i''	0	$\frac{30}{h^2}$	$\frac{270}{h^2}$	$\frac{-300}{h^2}$	$\frac{-300}{h^2}$	$\frac{270}{h^2}$	$\frac{30}{h^2}$	0

3. Notations, assumptions and definitions

In this section we present several notations and assumptions that will be used in the sequel. We use the standard functional spaces $L_2(\Omega)$, $V = \{\upsilon; \upsilon \in W_2^{(k)}(\Omega)\}$ on Γ the sense of traces,

$C(I; L_2(\Omega))$, $L_2(I; L_2(\Omega))$ (see e.g. [14], [15]). By $(.,.)$ we shall denote either the inner product in $L_2(\Omega)$ or the duality between V and V^* (dual of V). We denote by $|.|$, $\|.\|$, $\|.\|_*$, the. norms in $L_2(\Omega)$, V, and V^*, respectively. All the constants which occur in the course of this paper will be denoted by C (ε) is small and $C_\varepsilon = C(\varepsilon^{-1})$). Also, we introduce some notations concerning the time discretization of our problem.

$$\delta z_i = \frac{z_i - z_{i-1}}{\tau}, \qquad \bar{z}^i = \frac{1}{\tau}\int_{I_i} z(.,t)dt, \qquad 1 \le i \le n, \tag{3.1}$$

For any given family $\{z_i\}_{i=0}^n$. The following elementary relations will be used in the following analysis:
For positive constants K and β exist such that the inequalities

$$|((\upsilon, u))| \leqq K \|\upsilon\|_{W_2^{(k)}(\Omega)} \|u\|_{W_2^{(k)}(\Omega)} \quad \text{for all} \quad \upsilon, u \in W_2^{(k)}(\Omega), \tag{3.1}$$

$$((\upsilon, \upsilon)) \geqq \beta \|\upsilon\|_V^2 \qquad \text{for all } \upsilon \in V \tag{3.2}$$

We will assume, throughout this work, the following hypotheses on the given data.

(H1) The kernel $K(x,t,u)$ is Lipschitz continuous in the variables t and u in the following sense:

$$\|K(x,t_2,u_2) - K(x,t_1,u_1)\| \leqq C(|t_2 - t_1| + \|u_2 - u_1\|), \tag{3.3}$$

and it satisfies

$$\forall t \in I = [0,T], u \in V \Rightarrow K \in L_2(\Omega) \tag{3.4}$$

(H2) $u_0 \in V \Rightarrow u_0 \in L_2(\Omega)$
Under these assumptions, we can define the variational solution of problem (1.11)-(1.13).

Definition 2.1 The measurable function $u \in L(I,V) \cap C(I, L_2(\Omega))$, with $u'(t) \in L_2(I, L_2(\Omega))$, and $u(0) = 0$ in $C(I, L_2(\Omega))$ is said to be a weak (variational) solution of (1.11)–(1.13) if and only if the integral identity

$$\int_0^T ((\upsilon, u))dt + \int_0^T (\upsilon, u')dt = \int_0^T (\upsilon, W)dt + \int_0^T (\upsilon, f)dt \tag{3.4}$$

holds for all $\forall \upsilon \in L_2(I,V)$.

4. **The semidiscretization scheme. A prioi estimates**

Our main goal is to approximate (1.11)–(1.13) from a numerical point of view and to prove its convergence. The suggested technique is based on the combination of the characteristics and Roth methods. Using a 2-point Euler backward differentiation formula for the time derivative and then applying the characteristic method to compensate the convection term which is discretized explicitly so that the underlying equation is converted into a linear system of algebraic equations that easily solved numerically at each subsequent time level. To this purpose, let n be a positive integer. Subdivide the time interval I by the points t_i, where $t_i = i\,\tau$, $\tau = T/n$, $i = 0, 1, \cdots, n$. The suggested discretization scheme of problem (2.5) consists of the following problem (in the weak sense):

Find $z_i \cong u(., t_i) \in V,\quad i = 1, \cdots, n$ such that

$$z_0 = u_0 \qquad \text{in } \Omega \tag{4.1}$$

$$z_i^*(x) = \tilde{z}_i(x - \tau\alpha) \tag{4.2}$$

$$z_{i+1} - \tau \frac{d^2 z_{i+1}}{dx^2} - z_i^* + \tau \int_0^{t_{i+1}} k_{i+1}(s)\, z_{i+1}(x, s) = \tau\ f_{i+1}(x) \tag{4.3}$$

where $z_{i+1} = z(x, t_{i+1}),\quad k_{i+1}(s) = k(t_{i+1}, s),$ and $f_{i+1}(x) = f(t_{i+1}, x).$

The later integral will be handled numerically using the composite weighted trapezoidal rule given by:

$$\begin{aligned}\int_{t_0}^{t_{i+1}} f(s)\, ds &\approx \tau \sum_{m=0}^{i} [w\ f(t_m) + (1-w)\ f(t_{m+1})] \\ &= \tau \left[w\ f(t_0) + (1-w)\ f(t_{i+1}) + \sum_{m=1}^{i} f(t_m) \right]\end{aligned} \tag{4.4}$$

Using (4.4) we get

$$\int_0^{t_{i+1}} k_{i+1}(s)\ z(x, s)\, ds \approx$$

$$\approx \tau \left(w\, k_{i+1}(0)\ z_0(x) + (1-w)\ k_{i+1}(t_{i+1})\ z_{i+1}(x) + \sum_{m=1}^{i} k_{i+1}(t_m)\ z_{i+1-m}(x) \right) \tag{4.5}$$

The substitutions of this equation into equation (4.3) yields

$$(1 + \tau^2 (1-w)\, k_{i+1}(t_{i+1}))\ z_{i+1}(x) - \tau \frac{d^2 z_{i+1}}{dx^2} =$$

$$= \tau\ f_{i+1}(x) + z_i^* - \tau^2 \left(w\, k_{i+1}(0)\ z_0(x) + \sum_{m=1}^{i} k_{i+1}(t_m)\ z_{i+1-m}(x) \right) \tag{4.6}$$

5. The Spline-Collocation Method

In this section a spline method for solving (4.6) is outlined, which is based on the collocation approach. Let $Z_i(x)$ be a function that approximates $z(x, t_i)$ for the time-level $t_i = i\,\tau,$ and is a linear combination of n+1 shape functions which is expressed as:

$$Z_i(x) = \sum_{m=0}^{n} c_{mi}\ B_m(x), \tag{5.1}$$

where $\{c_{mi}\}_{m=0}^{n}$ are the unknown real coefficients, to be evaluated, and the $B_m(x)$ are sixth-degree B-spline. The approximate solutions $z_i(x)$ for different time-levels are determined iteratively as follows. Starting with the time-level $t_0 = 0$, the value of $z_0^*(x_j)$, and $z_0(x_j)$, for $j = 1, 2, \ldots, n-1$ are known. Next, we will approximate the solution z_{i+1} for i = 0 in equation (4.6) by the shape functions Z_1, as is given in equation (5.1). Hence equation (4.6) is approximated by:

$$(1+\tau^2(1-w)\,k_1(t_1))\,Z_1(x) - \tau\,\frac{d^2 Z_1}{dx^2} = \tau\, f_1(x) + z_0^* - \tau^2\, w\, k_{i+1}(0)\, z_0(x). \qquad (5.2)$$

Replacing Z_1 by the approximate solution given by equation (5.1) yields the following linear system of $n-1$ equations

$$\sum_{m=0}^{n} c_{m1}\Big((1+\tau^2(1-w)\,k_1(t_1))\,B_m(x) - \tau\, B_m''(x)\Big) = \tau\, f_1(x) + z_0^* - \tau^2\, w\, k_{i+1}(0)\, z_0(x). \qquad (5.3)$$

Putting $x = x_j, \quad j = 1, \cdots, n-1$, where

$$a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b, \qquad x_{j+1} - x_j = h,$$

In such a case we have $x_j = a + j\,h$ for $j = 0, 1, 2, \ldots n$, so equation (5.3), rewrite as

$$\sum_{m=0}^{n} c_{m1}\Big((1+\tau^2(1-w)\,k_1(t_1))\,B_m(x_j) - \tau\, B_m''(x_j)\Big) = \tau\, f_1(x_j) + z_0^* - \tau^2\, w\, k_{i+1}(0)\, z_0(x_j). \qquad (5.4)$$

The system (5.4) consists of $(n-1)$ equation in the $(n+1)$ unknowns $\{c_{m1}\}_{m=0}^{n}$. To get a solution of this system we need two additional conditions. These conditions are obtained from the boundary conditions (1.12)

$$z(a,t_i) = \sum_{m=0}^{n} c_{m1}\, B_m(a) = g_1(t_i), \qquad i = 0, \ldots n \qquad (5.5)$$

$$z(b,t_i) = \sum_{m=0}^{n} c_{m1}\, B_m(b) = g_2(t_i), \qquad i = 0, \ldots n. \qquad (5.6)$$

The system (5.4), equations (5.5) and (5.6) consist of $(n+1)$ equations in $(n+1)$ unknowns; this system is of the form

$$AC = F. \qquad (5.7)$$

Upon solving the system (5.7), the function $Z_1(x)$ is approximated by the sum:

$$Z_1(x_j) = \sum_{m=0}^{n} c_{m1}\, B_m(x_j), \; j = 0, 1, 2, \ldots, n. \qquad (5.8)$$

Next, we find the approximate solution at time-levels $t_1, t_2, \ldots$ recursively by solving the following system for $i = 1, 2, \ldots$.

$$\sum_{m=0}^{n} c_{mi}\Big((1+\tau^2(1-w)\,k_{i+1}(t_{i+1}))\,B_m(x) - \tau\, B_m''(x)\Big) =$$

$$= \tau\, f_{i+1}(x) + z_i^* - \tau^2\, w\, k_{i+1}(0)\, z_0(x) - \tau^2 \sum_{m=1}^{i} k_{i+1}(t_m)\, z_{i+1-m}(x). \tag{5.9}$$

$$z(a,t_i) = \sum_{m=0}^{n} c_{mi}\, B_m(a) = g_1(t_i), \qquad i = 0,\dots n \tag{5.10}$$

$$z(b,t_i) = \sum_{m=0}^{n} c_{mi}\, B_m(b) = g_2(t_i), \qquad i = 0,\dots n. \tag{5.11}$$

6. Numerical Results

In this section, we shall solve integro-differential equation (1.11) in $Q_T \equiv (0,1)\times(0,T)$. We employ an explicit central difference scheme for the space derivative so that we get a full discretization scheme with an error estimation $O(h^2)+O(\tau)$. The boundary and initial conditions we have used in this experiment are $u(0,t)=u(1,t)=0,\quad t<0$ and $u(x,0)=\sin(\pi x)\quad 0\le x\le 1$ for which the theoretical solution is $u(x,t)=e^{-\pi^2 t}\sin(\pi x)$, and $f(x,t)=-t\,e^{-\pi^2 t}\sin(\pi x)$ and $k(x,t)=e^{-\pi^2(x-s)}$ We shall compare the results obtained by the suggested approximation scheme (3.1)-(3.3) with the exact. It is observed that all the results of the proposed approximation scheme are in good agreement with the exact ones and exhibit the expected convergence.

Table 2. Comparison between exact and numerical solutions at $t=0.02,\ \alpha=1,\ \tau=0.0001$ and $t=0.01,\ \alpha=1,\ \tau=0.0001$, respectively.

x	$t=0.02,\ \alpha=1\ \ \tau=0.0001$			$t=0.01,\ \alpha=1\ \ \tau=0.0001$		
	Exact solution	Suggested scheme	error	Exact solution	Suggested scheme	error
0	0	0.000000	0	0	0	0
0.17	4.104E-001	4.111E-001	6.353E-004	4.530E-001	4.537E-001	7.016E-004
0.33	7.108E-001	7.115E-001	6.192E-004	7.846E-001	7.853E-001	6.842E-004
0.5	8.208E-001	8.214E-001	5.702E-004	9.060E-001	9.066E-001	6.303E-004
0.67	7.108E-001	7.114E-001	5.3021E-004	7.846E-001	7.852E-001	5.859E-004
0.83	4.104E-001	4.108E-001	4.196E-004	4.530E-001	4.535E-001	4.635E-004
1	0	0	0	0	0	0

Table 3. Comparison between exact and numerical solutions at $t=0.1,\ \alpha=1,\ \tau=0.00001$ and $t=0.5,\ \alpha=1,\ \tau=0.01$, respectively.

x	$t=0.1,\ \alpha=1,\ \nu=1,\ \tau=0.00001$			$t=0.5,\ \alpha=1,\ \tau=0.01$		
	Exact solu-	Suggested	error	Exact solu-	Suggested	error

	tion	scheme		tion	scheme	
0	0	0	0	0	0	0
0.17	1.864E-001	1.864E-001	2.873E-005	3.596E-003	4.316E-003	7.196E-004
0.33	3.228E-001	3.228E-001	2.786E-005	6.228E-003	6.794E-003	5.660E-004
0.5	3.727E-001	3.727E-001	2.559E-005	7.192E-003	7.693E-003	5.013E-004
0.67	3.228E-001	3.228E-001	2.380E-005	6.228E-003	6.696E-003	4.672E-004
0.83	1.864E-001	1.864E-001	1.888E-005	3.596E-003	3.987E-003	3.912E-004
1	0	0	0	0	0	0

Table 4. Comparison between exact and numerical solutions at $t = 0.3$, $\alpha = 0.4$, $\tau = 0.00005$ and $t = 0.7$, $\alpha = 3$, $\tau = 0.04$, respectively.

x	$t = 0.3$, $\alpha = 0.4$, $\tau = 0.00005$			$t = 0.7$, $\alpha = 3$, $\tau = 0.04$		
	Exact solution	Suggested scheme	error	Exact solution	Suggested scheme	error
0	0	0	0	0	0	0
0.17	2.589E-002	2.591E-002	1.816E-005	4.995E-004	1.618E-002	1.568E-002
0.33	4.484E-002	4.486E-002	1.818E-005	8.652E-004	1.822E-003	9.572E-004
0.5	5.177E-002	5.179E-002	1.729E-005	9.990E-004	7.164E-004	2.826E-004
0.67	4.484E-002	4.485E-002	1.694E-005	8.652E-004	7.362E-004	1.290E-004
0.83	2.589E-002	2.590E-002	1.491E-005	4.995E-004	5.472E-004	4.767E-005
1	0	0	0	0	0	0

7. Conclusion

A collocation B-spline method has been considered for the numerical solution of integro-partial differential problems. The sixth-degree B-spline method was tested by problem. The method reduces the underlying problem to linear system of algebraic equations, which can be solved successively to obtain a numerical solution at varied time-levels. Numerical experiments which shown in the above scheme are good agreement with the exact ones. Moreover, the results in tables 1-4 and confirm that the numerical solutions can be refined when the time-step τ is reduced, or the number of nodes is increased.

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