The Form of the Optimal Control of Semilinear Dynamical Systems with Multiple Delays in the Control

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Abstract. In this paper, finite-dimensional, stationary dynamical control systems described by semi linear ordinary differential state equation with multiple delays in control of the form

$$\dot{x}(t) = A\,x\,(t) + f\,(x,t) + \sum_{j=0}^{j=m} \mathrm{B_j u}\left(\mathrm{t} - \mathrm{h_j}\right) (2.1)$$

is presented for controllability analysis. Our principal objective is to establish the form of optimal control of the class of systems described by the system (2.1) .For purposes of clarity; we define the following terminologies-*attainable set, reachable set , Target set* upon which our study hinges. The attainable, reachable and target sets of the system (2.1) are extracted from the solution of the system (2.1). Uses are made of theorm2.1 andsignum function to establish the form of the optimal control and/or result.

Key words: Signum function multiple delays, semilinear state equation, attainable set, target set and reachable set.

1.0 INTRODUCTION

Systematic study of controllability started over the years at the beginning of the sixties when the theory of controllability based on the description in the form of state space for both time-varying and time-invariant linear control systems was carried out. Roughly speaking, controllability generally means that, it is possible to steer a dynamical control system from an initial state to a final state using the set of admissible controls. Optimal control means doing the same in the best conceivable way.

There are many different definition of controllability which strongly depend on the class of dynamical control systems. In the recent years, various controllability problems for different types of nonlinear systems have been considered. However, it should be stressed that, most of the reported work with is direction has been mainly concerned with controllability for linear dynamical systems with constrained control and without delays (see Klamka (1991), Sun (1996), under wood and young (1979),

Later on delay differential equations came to limelight (see Nse (2007), Nse (2008). A delay equation on a linear system is one which affects the evolution of the system in an indirect manner.

If we consider the equation

$$\dot{x} = A\,x(t) + Bu(t),$$

Where A and B are nxn and nxm matrices, we see that the action of the control is direct in that the local bebahviour of the trajectory is affected only by the local behavior of the control u(t) at time t. It is known that, most of the natural applications give rise to mechanism of indirect actions where decisions in the control function are shifted, twisted or combined before affecting the evolution thus composing the delay u(t-h) represented by the system.

$\dot{x} = Ax(t) + Bu(t-h)$

In this paper, we shall consider optimal control format for finite-dimensional stationary semilinear dynamical systems with multiple points delays in the control described by ordinary differential state equations. Let us recall, that semilinear dynamical control systems contain linear and pure nonlinear parts in the differential state equations (see K. Naito (1987). We shall derive the form of the optimal control of our system of interest and express same using the definition of the signum function

Definition 1.0: (Signum)

The signum function is defined by

$$sgn(x) = \begin{cases} 1, if\ x > 0 \\ -1, if\ x < 0 \end{cases}$$

Description of the System of Interest

In this work, we study the semilinear stationary finite-dimensional dynamical control system with multiple point delays in the control described by the following ordinary differential state equation of the form

$$\dot{x}(t) = Ax(t) + f(x,t) + \sum_{j=0}^{j=m} \mathrm{B_j u}(\mathrm{t} - \mathrm{h_j}) \qquad (2.1)$$

$For\ t \in (0, t,), t > h, with\ zero\ initial\ conditions\ x(0) = 0;\ u(t) = 0\ for\ t\in[-h, 0] \ldots (2.2)$

$Where\ the\ state\ x(t)\in R^n = X\ and\ the\ control\ u(t)\in R^m = U,$

$A\ is\ nxn\ dimensioned\ constant\ matrix, B_{j,}(j = 0, 1, 2, \ldots, m) are\ nxm\ dimensional\ constant\ matrices, 0 = h_0 < h_1 < \cdots < h_j < \ldots < h_m = h$ are constant delays. Moreover, let us assume that the nonlinear mapping F:X → X is continuously differentiable near the origin and such that F(0) = 0. Then the set of admissible controls for the dynamical control systems (2.1) has the following form U = L∞ ([0,t,], U_c), where U_C is closed and convex cone with nonempty interior and vertex at zero. Then for any given admissible control u(t) there exists a unique solution x(t,u) for t∈ [0, t,], of the state equation (2.1) with zero initial conditions (2.2) described by the integral formula (see Klamka (2004)).and J.U.Onwuatu (1988).

$$x(t,u) = \int_0^t G(t-s)\left[f(x(s,u)) + \sum_{j=0}^{j=m} \int_{t_0-h_j}^{t_0} \mathrm{X}(\mathrm{t_0}, \mathrm{s}+\mathrm{h_j})\mathrm{B_j}(\mathrm{s}+\mathrm{h_j})\mathrm{u_0}(\mathrm{s})\mathrm{ds}\right.$$

$$\left. + \sum_{j=0}^{m} \int_{t-h_{j+1}}^{t-h_j} \sum_{p=0}^{j} X(t_0, s+h_p)\, B_p(s+h_p) u(s)\, ds \right] \qquad (2.3)$$

$satisfying\ x(t_0) = x_0\ with, h_m > h_{m-1} > \cdots > h_1 > h_0 = 0$

$Where\ the\ semigroup\ G(t) = \exp(At)\ is\ nxn\ transition\ matrix\ for\ the\ linear\ part\ of\ the\ seminear\ control\ system\ (2.1).$

For the semilinear dynamical system (2.1) with multiple delays in the control, we shall focus our attention on the so called form of optimal control in the interval $[0, t_1]$.

In order to do this, we first all introduce the notions of: $\boldsymbol{attainable\ set\ A\ (t_1, 0)}$, $\boldsymbol{reachable\ set\ R\ (t_1, 0). Target\ set\ G\ (t_1, 0)}$ $at\ time\ t_1 > 0$ $from\ zero\ initial\ conditions\ (2.2)\ and\ defined\ as\ follows$;(see J. Klamka (2004), T.I. Seidman (1987)).

Definition 2.1: Attainable Set

$$A(t_1, 0) = \{x\epsilon\, R^n : x = x\,(t_1, u) : u(t)\,\epsilon U\ for\ a.e: t\epsilon\,[0, t_1\,]\,\} \qquad (2.4)$$

$Where\ x(t\,, u) : t\ > 0$ is the unique solution of the equation (2.1) with zero initial conditions (2.2) and a given control u. Under the assumptions stated on the nonlinear term F such solution always exists [see H.X. Zhou (1984) and K. Naito (1987)].

Now, using the concept of the attainable set, let us define the other terminologies upon which our study hinges:

Definition 2.2: Reachable set

The reachable set for the system (2.1) is given as

$$R(t_1, 0) = \left\{ \int_0^t G\,(t-s) \sum_{j=0}^{m} \int_{t-h_{j+1}}^{t-h_j} \sum_{p=0}^{j} X\left(t_0, s + h_p\right) B_p\left(s + h_p\right) u(s)\, ds\ ] \right\}$$

Definition 2.3: Target Set

$The\ target\ set\ for\ system\ (2.1)\ denoted\ by\ G(t_1, 0), given\ as$

$G(t_1, 0) = \{x(t_1, u) : t_1 > \tau > t_0\ for\ some\ fixed\ \tau\ and\ u\epsilon\ U\}.$

We shall derive the form of the optimal control of our system (2.1) and express same using the definition of the signum function.

Theorem 2.1

Consider the semilinear stationary finite-dimensioned Dynamical control system with multiple point delays in the control described by the following ordinary differential state equation.

$$x(t) = A\,x\,(t) + F\,(x, t) + \sum_{j=0}^{j=m} \mathrm{B_j\,u}\left(\mathrm{t} - \mathrm{h_j}\right) (2.1)$$

$with\ the\ basic\ assumptions, u^*\ is\ the\ optimal\ control\ energy\ for\ system\ (2.1) if\ and\ only\ if\ u^*\ is\ of\ the\ form$

$$u^*(t) = sgn\left(C^T \int_0^t G\,(t-s) \sum_{j=0}^{m} \int_{t-h_{j+1}}^{t-h_j} \sum_{p=0}^{j} X\left(t_0, s + h_p\right) B_p\left(s + h_p\right) u(s)\, ds \right),$$

$where\ C\epsilon R^n = X.$

Proof

- ⇨ Suppose u^* is the optimal control energy for system (2.1), then it maximizes the rate of change of

$$y\,(t, u) = \int_0^t G\,(t-s) \sum_{j=0}^{m} \int_{t-h_{j+1}}^{t-h_j} \sum_{p=0}^{j} X\left(t_0, s + h_p\right) B_p\left(s + h_p\right) u(s)\, ds, for\ u\epsilon U$$

in the direction of C.

Since u(t) is an admissible control, that is it is constrained to lie in a unit sphere, we have

$$C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u(s)ds$$
$$\leq \left|C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p).1ds\right|$$
$$\leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds$$
$$sgn\; C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds \qquad (2.5)$$

This inequality follows from the fact that for any non $-$ *zero number* $N, N \leq sgn\, N$.

Hence defining

$$u^* = Sgn\; C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds \qquad (2.6)$$

The inequality (2.5), becomes

$$C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u(s)ds$$
$$\leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u^*(s)ds$$
$$\leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p).1ds, \textit{since for } N\neq 0, sgnN > 0$$

this shows that the control that maximizes $y(t,u)\,\epsilon R(t_1,0)$ *is of the form*

$$u^* = Sgn\left[C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds\right]$$

Conversely,

Let $u^* = Sgn\left[C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds\right]$, *then for the*

Controls $u\,\epsilon\, U$,

$$C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u(s)ds$$
$$\leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds$$
$$sgn\; C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)ds$$

$$\leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p).1ds, since\ for\ N\neq 0, sgnN$$

$$> 0. \leq C^T\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u^*(s)ds$$

This shows that u^* *maximizes*

$$\int_0^t G(t-s)\sum_{j=0}^{m}\int_{t-h_{j+1}}^{t-h_j}\sum_{p=0}^{j}X(t_0,s+h_p)\,B_p(s+h_p)u(s)ds, for\ u\epsilon U$$

over all *admissible controls* $u\,\epsilon U$.

Hence u^* *is an optimal control for system* (2.1). *this completes the proof*.

CONCLUSION

We have established the form of optimal control **of Semilinear Dynamical Systems with Multiple Delays in the Control**and expressed same using the **Definition of Signum Function and Theorem 2.1**

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Relative Controllability of Functional Differential Systems of Sobolev Type in Banach Spaces

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Abstract. In this paper, the relative controllability of functional differential systems of Sobolev Type in Banach Spacesof the form

$$(Ex(t))' + Ax(t) = Bu(t) + f(t, x_t), t > 0 \qquad (1.1)$$

was presented for controllability analysis. For purposes of clarity, we defined and extracted the following terminologies as they relate to system (1.1) from the solution given as integral formula [system (1.2)] **- complete set, reachable set, attainable set, target set , properness and relative controllability**. Necessary and sufficient conditions for the system (1.1) to be relatively controllable are established using Theorem 2.1 and Theorem 2.2. That is, (a) The system is relatively controllable if and only is zero is in the interior of the reachable set(b) The system is relatively controllable if the controllability grammian (map) of system (1.1) is non-singular.The results are established using the controllability standard-properness. The reachable set, attainable set and target set upon which our results hinge are extracted from the system (1.2) or the solution of system (1.1).

Keywords: Relatively Controllable, Relative Controllability, Functional Differential Systems, Sobolev Spaces, Banach Spaces. Reachable Set, Attainable Set and Target Set.

1.INTRODUCTION

The controllability of functional differential systems of Sobolev type in Banach Spaces had been established by KRISHNAN BALACHANDRAN AND JERALD DAUER(1998).

According toKrishnan Balachandran And Jerald Dauer(1998),the problem of controllability of linear and nonlinear systems represented by ordinary differential equations infinite dimensional spaces has been extensively studied. Several authorizes (see E.N. Chukwu (1991), R.F. curtain and A.J. Prichard (1978), I. Lasieka and R. Triggiani (1991) have extended the concept of infinite dimensional systems in Banach spaces with bounded operators. R. Triggiani (1975) established sufficient conditions for controllability of linear and nonlinear systems in Banach spaces with bounded operators. R. Triggiani (1975) established sufficient conditions for controllability of linear and nonlinear systems in Banach Spaces. Exact controllability of abstract semilinear equations has been Studied by I. Labiecka and R. Triggiani (1991). Y.C. Kwun, T.Y. Park and J.W. Ryu (1991) investigated the controllability and approximate controllability of delay volterra systems by using fixed point theorem. K. Balachandran, P. Balasubramaniam and J.P. Daner (1995), K. Balachandran, P. Balasubramaniam and J.P. Dauer (1995) and K. Balachandran, P. Balasubramaniam and J.P. Dauer (1996) studied the controllability and local null controllability of nonlinear integro-differential systems and functional differential systems in Banach Spaces and it was shown that controllability problem in Banach Spaces can be converted into one of a fixed-point problem for a single le-valued mapping. It is known from Onwuatu, J.U. (1993) that if a system is relatively controllable, the optimal control is unique and Ban-Bang. In the light of this, we shall consider the functional differential systems of Sobolev Type in Banach Spaces of the form.

$$(E\,x\,(t))\,' + A\,x\,(t) \;=\; Bu(t) + f(t, x_t), t \;>\; 0 \qquad (1.1)$$

(a nonlinear partial functional differential system) where the state x(.) takes values in a Banach Spaces X and the control function u(.) is given in L^2(J, U), the Banach Spaces of admissible control functions with U a Banach Space. B is a bounded linear operator from U into Y, a Banach Space. The nonlinear operator F: JxC → Y is continuous. Here J = [0, t_1] and for a continuous function $x: J* = [-l, t_1] \to X, x_t$ *is that element of* $C = C([-l, 0]: X)$ *defined by*
$x_t(s) \;=\; x\,(t+s);\; -\iota \le\; s \;\le\; 0.$
The domain of E D(E) becomes a Banach Space with norm
$||x||_{D(E)} \;=\; ||\,Ex||_Y, x\epsilon D(E)$ *and* $C(E) \;=\; C\;([-\iota, 0]: D(E).$

The above system (1.1) will be investigated for **relative controllability, existence, form and uniqueness of optimal control** by first of all considering the relative controllability of the system. For a given admissible control u(t) there exists a unique solution $x(t, u)$ *for* $t \,\epsilon\, (0, t_1)$ of the system (1.1) described by the integral formula , see Krishnan Balachandran and Jerald P. Dauer (1998).

$$x(t) \;=\; E^{-1}T(t)\,E\Phi(0) + \int_0^t E^{-1}\mathrm{T}(t-s)f\;(s, x_s)ds + \int_0^t E^{-1}\mathrm{T}(t-s)Bu(s)ds$$
$$x(t) = \Phi(t), -\iota \;\le t \;\le 0. \qquad (1.2)$$

For purposes of clarity, we define the following terminologies as they relate to system (1.1). With the solution given as integral formula [system (1.2)], we define **complete set, reachable set, attainable set, target set , properness and relative controllability**.

Definition 1.1 (complete state)

The complete state for system (1.1) is given by the *set*
$\boldsymbol{z(t) \;=\; \{x, u_t\}}$

Definition 1.2 (Reachable Set)

The reachable for the system (1.1) is given as

$$\boldsymbol{R(t_1, 0) = \left\{\int_0^t E^{-1}T(t,s)\,Bu(s)ds\right\}}$$

Definition 1.3 (Attainable Set)

Attainable set is the set of all possible solutions of a given control system. In the case of the system (1.1), for instance, it is given as

$$\boldsymbol{A\left(t_{1,}0\right),}$$
$$\boldsymbol{= \left\{x\;(t) \;=\; E^{-1}T\;(t)\,E\;\Phi(0) + \int_0^t E^{-1}T\;(t-s)\;f(s, x_s)ds + \int_0^t E^{-1}T\;(t-s\;B(s)u(s)ds;\; u\epsilon U\right\}}$$

$$\boldsymbol{= E^{-1}\left\{T\;(t)[\mathrm{E}\Phi(0)] + \int_0^t T\;(t-s)\;f(s, x_s)ds + \int_0^t T\;(t-s\;B(s)u(s)ds\;; u\epsilon U\right\}}$$
$$\boldsymbol{= \{\eta + R\;(t_1, 0)\}.}$$
$$\boldsymbol{where\;\eta = E^{-1}T\;(t)\,E\;\Phi(0) + \int_0^t E^{-1}\,T\;(t-s)f(s, x_s)ds}$$

Since E (t) is a fundamental matrix and fundamental matrices are invertible, E^{-1} *exists.*
Or

Attainable Set for the system (1.1) is given as

$\mathbf{A}\ (t_1, 0) = \{x(t,u): u\epsilon U\}$, *where* $U = \{u\epsilon L_2\ ([0,t_1], X): |u_j| \leq 1\ ;\ j = 1,2,\ldots,m\}$.

Definition 1.4(Target Set)

The target set for system (1.1) *denoted by* $G\ (t_1, 0)$ *is given as*

$G\ (t_1, 0) = \{x(t) = x\ (t,u): t_1 \geq \tau > 0$ *for fixed* τ *and* $u\epsilon U\}$.

Definition 1.5 (Properness)

The system (1.1) *is proper in* X *on* $(0, t_1)$ *if and only if.*

$C^T[E^{-1}T\ (t-s)B(s)] = 0$ (*almost every where*) $t_1 > 0 \Rightarrow C = 0, C\epsilon X$.

Definition 1.6 (Controllability Grammian)

The controllability grammian for the system (1.1) is given as

$$W\ (t_1, 0) = \int_0^t z(t,s)\, z^T(t,s) ds$$

Where $z(t,s) = [E^{-1}T(t-s)B(s)]$ *and* T *denotes matrix transpase .*

Definition 1.7 (Relative Controllability)

The system (1.1) *is said to be relatively controllable on* $[0, t_1]$ *if for every initial complete state* $z(0)$ *and* $x_1 \epsilon X$, *there exists a control function* $u(t)$ *defined in* $[0, t_1]$ *such that the solution* (1.2) *of the system* (1.1) *satisfies* $x(t_1) = x_1$.

2. Main Results

We now state and prove the following theorems that guarantee relative controllability of the system (1.1) under study.

Theorem 2.1.(Necessary Condition).

consider the system (1.1) given as

$$(E\,x(t))^{1} + A\,x(t) = Bu(t) + f(t, x_t), t > 0 \qquad (2.1)$$

With the same conditions on the system parameters as in (1.1), then the following statements are equivalent:

(1) ***System* (1.1) *is relatively controllable on*** $[0, t_1]$

(2) ***The controllability grammian*** $W(t, 0)$ ***of system* (1.1) *is nonsingular.***

(3) ***The system* (1.1) *is proper on*** $[0, t_1]$.

Proof

Straight forward from the arguments in, Anajevskii and Kolmanovskii (1990), Angell (1990), Balachandran and Dauer (2002).

Theorem 2.2 (Sufficient Condition)

The system (1.1) with its standing hypothesis is relatively controllable if and only if zero is in the interior of the reachable set. That is system (1.1) is relatively controllable if and only if.

$$0 \epsilon Int\ R\ (t_1, 0) for\ t_1 > 0.$$

Proof

The reachable set $R(t_1, 0)$ *is a closed ,convex and compact subset of* X. Therefore, a *point* $z_1 \epsilon X$ *on the boundary implies there is a support plane* π *of* $R(t_1, 0)$ *through* z_1.

That is,

$$C^T(z - z_1) < 0 \text{ for each } z \epsilon R(t_1, 0),$$

where $C \neq 0$ *is an outward normal to the support plane* π.

If u_1 *is the corresponding control to* z_1, *we have*

$$\mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s)] u(s) ds \leq \mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s)] u_1(s) ds, \text{ for each } u \epsilon U$$

(2.2)

Since U is a sphere, the inequality (2.2) becomes

$$\left| \mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s)] u(s) ds \right| \leq \left| \mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s)].1 \right| \mathrm{ds}$$

$$= \mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s).1] \mathbf{sgn}\, \mathrm{C^T} [\mathrm{E}^{-1} T(t-s) B(s)] ds \qquad (2.3)$$

Compare equation (2.2) with equation (2.3) we have

$$u_1(t) = sgn \mathrm{C^T} [\mathrm{E}^{-1} T(t-s) B(s)]$$

More so, as z_1 *is on the boundary, since we always have* $0 \epsilon R(t_1, 0)$.

If 0 *were not in the interior of* $R(t_1, 0)$, *then it is on the boundary*.

Hence, from the preceding argument it implies that

$$0 = \mathrm{C^T} \int_0^{t_1} [\mathrm{E}^{-1} T(t-s) B(s)] \, \mathrm{ds}$$

So that

$\mathrm{C^T}[\mathrm{E}^{-1} T(t-s) B(s)] = 0$, almost everywhere.

Then, by the definition of properness, this implies that the system is not proper, *since* $C^T \neq 0$.

However, if 0ϵ *Interior* $R(t_1, 0)$ *for* $t_1 > 0$

$$C^T[E^{-1} T(t-s) B(s)] = 0, \Rightarrow C = 0,$$

which is the properness of the system (1.1) and by the equivalence in theorem 2.1, the relative controllability of the system (1.1) on the interval $[0, t_1]$ *is proved*.

CONCLUSION

We have established Necessary and Sufficient Conditions for Functional Differential Systems of Sobolev Type in Banach Spaces to be Relatively Controllable, using the Controllability Standards-(Properness of the Systems, and Zero being in the interior of the Reachable Set) and theorem 2.1 / theorem 2.2.

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