New view of multisets relations

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Abstract. Multiset is a collection of objects in which repetition of elements is significant. In many real life problems, medical investigation and teaching for example, the repetition of cases affect**s** the process of decisions making, and so the multiset is a suitable theory for modeling such cases. In this paper we initiate a matrix representation for relations on multisets and given examples. Also a generalization for composition of relation is defined and its properties are studied, many examples are given. After and for sets for multisets are studied. These suggested relation can help in constructing new connections for multisets with information systems whose objects are multisets.

Keywords: Multiset- Multi relation-Fuzzy set

1. Introduction

Many theories of modern mathematics have emerged by violating a basic principle of a given theory of traditional mathematics. Ordinary set theory implicitly assumes that all mathematical objects occur without repetition. Thus there is only one number four, one field of complex numbers, etc. So the only possible relation between two mathematical objects is either they are equal or they are different. The situation in science and in ordinary life is not like this. In the physical world it is observed that there are enormous cases for repetition [Chakrabarty 1999, 2000, 2007]. For instance, there are many hydrogen atoms [Jena2001, Singh2003, Syropouios2001, and Yager1986], many water molecules, many strands of DNA, etc. In classical set theory, a set is a well-defined collection of distinct objects. Consequently, a new theory for modeling such cases has been appeared, this theory is the multiset (mset or bag, for short) [Manna 1985].

A multiset is a finite set in which each element is assigned by a positive integer called the number of repeated, so to represent a multiset we use a finite sequence of letters or objects. Some chemical terminology form by multiset as chemical soup of molecules and communication is viewed as a chemical reaction between molecules [Banatre1993]. Multiset is used in some type of graph transformation and in DNA computing. If we replace tubes of molecules by rules of applications we can study DNA by using multiset concepts [Goldbreg2002]. The important property of multiset is the repetition of its elements which is not found in classical set theory, since the possible relation between elements in classical set is either they are equal or they are different. The

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notion of sequential computation is important part in design of most programming language such that a good forms of abstraction of algorithms matching for preparing the desired result. So it is depends on multiset transformers [Hamkin1998].

Relation on set philosophy is based on the assumption that objects in set have a similar amount of knowledge or information expressed by relation. We can obtain hidden pattern in knowledge by using concept of relation on set of objects such that granulation of data, obtain lower and upper approximation of any subset of objects by using all granules sets [Yao2007].

Equivalent relation in rough set theory and rough multiset theory seems to be a stringent condition, so study of multiset relation may be extended too many applications of rough set theory and topological spaces on multiset.

In previous works, the generalization of the indiscernibility relations has been discussed in many classes of generalized relations [Yager1986].

Relation can be formulated based on the notions of fuzzy and rough set concepts [Greco2008]. Fuzzy and rough set theory attempts to provide an alternative interpretation of main parameters, the required parameters can be expressed in terms of different types of relations so; it is not difficult to establish connection between approximation structures and multiset relations.

The paper is organized as follows: We begin with the introduction to multiset and multiset relation in section 2. In section 3 and section 4, we attempt to generalize fuzzy relation and composition in multiset theory. New results on multiset relation are defined in section 5. Besides, several examples are given to indicate these definitions. At last, some conclusion is presented in section 6.

2. Preliminaries

Multiset is an unordered collection of objects, like a set but which may contain copies or duplicates [Brink1987]. In this section, we introduce a review of some basic concepts of multiset and multiset topology.

2.1. Multiset

Definition 2.1(Msets)[Jena2001**].** An mset M drawn from the set X is represented by the function $Count\ M$ or C_M, defined as $C_M: X \to N$, where N represents the set of nonnegative integers. For a positive integer n, the mset M drawn from the set $X = \{x_i\}_{i=1}^{n}$ is denoted by $M = \left\{\frac{k_i}{x_i}\right\}_{i=1}^{n}$, where $k_i = C_M(x_i)$ is the number of occurrences of the element x_i in the mset M. The elements which are not included in the mset M have zero count and an mset M is called an empty mset if $C_M(x) = 0\ \ \forall x \in X$.

Definition 2.2 [Jena2001**]** Let M and N be two msets drawn from a set X, and then the following definitions are defined.

- **(i)** The support set $M^* = \{x \in X:\ C_M(x) > 0\}$, and it is also called a root set.
- **(ii)** $M = N\ \ if\ \ C_M(x) = C_N(x)\ \ for\ all\ \ x \in X.$
- **(iii)** $M \subseteq N\ \ if\ \ C_M(x) \le C_N(x)\ \ for\ all\ \ x \in X.$
- **(iv)** $P = M \cup N\ \ if\ \ C_P(x) = max\{C_M(x),\ C_N(x)\}\ \ for\ all\ \ x \in X.$
- **(v)** $P = M \cap N\ \ if\ \ C_P(x) = min\{C_M(x),\ C_N(x)\}\ \ for\ all\ \ x \in X.$
- **(vi)** The cardinality of M is denoted by $Card(M)$ or $|M|$ and is given by

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$$Card(M) = \sum_{x \in M} C_M(x)$$

(vii) $P = M \oplus N \ \ if\ \ C_P(x) = C_M(x) + C_N(x)\ \ for\ all\ \ x \in X.$

$P = M \ominus N\ \ if\ \ C_P(x) = max\{C_M(x) - C_N(x), 0\}\ \ for\ all\ \ x \in X$, where $\oplus$ and $\ominus$ represent mset addition and mset subtraction, respectively.

Definition 2.3[Jena2001**]** A domain X is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times.

Definition 2.4 [Jena2001**]** Let X be a support set and $[X]^m$ be the mset space defined over X. Then for any mset $M \in [X]^m$, the complement M^c of M in $[X]^m$ is an element of $[X]^m$ such that $C_M^c(x) = m - C_M(x)\ \ \forall x \in X.$

Moreover, the following types of submset of M and collection of submsets from the mset space $[X]^m$ are defined.

Definition 2.5 (Whole submset) [Blizard1989**].** A submset N of M is a whole submset of M with each element in N having full multiplicity as in M, i.e.

$$C_N(x) =_{.} C_M(x)\ \ for\ every\ x\ in\ N^*.$$

Definition 2.6 (Partial Whole submset) [Blizard1989**].** A submset N of M is a partial whole submset of M with at least one element in N having full multiplicity as in M .i.e.

$$C_N(x) =_{.} C_M(x)\ \ for\ some\ x\ in\ N^*.$$

Definition 2.7(Full submset) [Blizard1989**].** Submset N of M is a full submset of M if each element in M is an element in N with the same or lesser non- zero multiplicity as in M, i.e.

$$M^* = N^*\ with\ C_N(x) =\leq_{.} C_M(x)\ \ for\ every\ x\ in\ N^*.$$

Example 2.2. Let $M = \left\{\frac{2}{x}, \frac{3}{y}, \frac{5}{z}\right\}$ be an mset, the following are the some of the submset of M which are whole submset, partial whole submset and full submset.

- A submset $\left\{\frac{2}{x}, \frac{3}{y}\right\}$ is a whole submset and partial whole submset of M.
- A submset $\left\{\frac{1}{x}, \frac{3}{y}, \frac{5}{z}\right\}$ is a partial whole and full submset of M.
- A submset $\left\{\frac{1}{x}, \frac{3}{y}\right\}$ is a partial whole submset of M.

As various subset relations exist in multiset theory, the concept of power mset can also be generalized as follow:

Definition 2.8 (Power whole Mset) [Blizard1989]. Let $M \in [X]^m$ be an mset. The power whole mset of M denoted by $PW(M)$ is defined as the set of all the whole submset of M The cardinality of $PW(M)$ is 2^n , where n is the cardinality of the support set of M.

Empty set φ is a whole submset of every mset but it is neither a full submset nor a partial whole submset of any nonempty mset M and the power whole multiset of any multiset is an $M - topology$.

Definition 2.9 (Power full Mset) [Blizard1989**].** Let $M \in [X]^m$ be an mset. The power full mset of M denoted by $PF(M)$ is defined as the set of all the full submset of M .The cardinality of $PF(M)$ is the product of the count of the element in M.

Definition 2.10 (Power Mset) [Jena2001**].** Let $M \in [X]^m$ be an mset. The power mset of M denoted by $P(M)$ is defined as the set of all the submset of M, i.e.

$N \in P(M)$ *if and only if* $N \subseteq M$

If $= \emptyset$ *then* $N \in {}^1P(M)$; *and if* $N \neq \emptyset$, *then* $N \in {}^k P(M)$, where $k = \prod_z \begin{pmatrix} |[M]_z| \\ |[M]_z| \end{pmatrix}$,

the product Π_z is taken over by distinct elements of z of the mset N and

$|[M]_z| = m \; iff \; z \in {}^mM$, $|[N]_z| = n \; iff \; z \in {}^nN$, then $\prod_z \begin{pmatrix} |[M]_z| \\ |[M]_z| \end{pmatrix} = \binom{m}{n} = \frac{m!}{n!(m-n)!}$

The power set of an mset is the support set of the power mset and is denoted by $P^*(M)$.

Theorem 2.1 [Jena2001**].** Let $P(M)$ be a power mset drawn from the mset $M = \{m_1/x_1, m_2/x_2, \dots, m_n/x_n\}$and $P^*(M)$.be the power set of an mset M. Then the card $P^*(M) = \prod_{i=1}^n (1 + m_i)$.

Example 2.3. Let $M = \{2/x, 3/y\}$ be an mset. The collection

$PW(M) = \{\{2/x\}, \{3/y\}, M, \emptyset\}$

$PF(M) = \{\{2/x, 2/y\}, \{2/x, 1/y\}, \{1/x, 3/y\}, \{1/x, 2/y\}, \{1/x, 1/y\}, M,\}$

$$P^*(M) = \left\{ \begin{matrix} M, \emptyset, \{2/x, 1/y\}, \{2/x, 2/y\}, \{1/x, 1/y\}, \{1/x, 2/y\}, \{1/x, 3/y\}, \{2/x\} \\ , \{3/y\}, \{1/x\}, \{1/y\}, \{2/y\} \end{matrix} \right\}$$

Definition 2.11 (Multiset topology) [Girish2012**].** Let $M \in [X]^m$ *and* $\tau \subseteq P^*(M)$, then τ is called a multiset topology if τ satisfies the following properties:

1. $\emptyset$ *and* M are in τ.
2. The union of the elements of any sub collection of τ is in τ.
3. The intersection of the elements of any finite sub collection of τ is in τ.

Note: The collection consisting of only M and $\emptyset$ is an M-topology called indiscrete M-topology

Note: If M is any mset, then the collection $PW(M)$ is an M-topology on M

Note: The collection $PF(M)$ is not an M-topology on M, because $\emptyset$ does not belong to $PF(M)$ but $PF(M) \cup \emptyset$ is an M-topology on M.

Definition 2.12 (Cartesian product) [Girish2012**]** Let M_1 and M_2 be two msets drawn from a set X, then the Cartesian product of M_1 and M_2 is defined as

$$M_1 \times M_2 = \{(m/x,\ n/y)/mn :\ x \in^m M_1,\ \ y \in^n M_2\ \}$$

Here the entry $(m/x,\ n/y)/mn$ in $M_1 \times M_2$ denotes x is repeated m times in M_1, y is repeated n times in M_2 and the pair $(x,\ y)$ is repeated mn times in $M_1 \times M_2$.

Definition 2.13 (Multiset relation) [Girish2012**]** Let M_1 and M_2 be two msets drawn from a set X.A relation R from M_1 to M_2 is a subset of $M_1 \times M_2$, i.e. $R \subset M_1 \times M_2$. For every $x \in^m M_1$, $y \in^n M_2$, the member $(m/x,\ n/y) \in R$ is abbreviated $(m/x)R(n/y)$ and has a $Count$, the product of $C_1(x,y)$ and $C_2(x,y)$,i.e., Thus the member $(x,\ y)$ of R is characterized by the number $R(x,y) = mn$.

3. New operations on multiset relations

In this section we will define the multiset on matrices and generalize the definitions of composition on multiset theory by considering some conditions on the counts of each multiset to satisfy the multi relation condition on multisets.

Definition 3.1(Multiset matrix): A multiset relation between elements in two finite sets

$X = \{m_1/x_1, m_2/x_2, \dots, m_k/x_k\}, Y = \{n_1/y_1, n_2/y_2, \dots, n_j/y_j\}$ can be represented as matrix

$$R = \begin{bmatrix} R(m_1/x_1, n_1/y_1) & \dots & R(m_1/x_1, n_j/y_j) \\ R(m_2/x_2, n_1/y_1) & \dots & R(m_2/x_2, n_j/y_j) \\ \dots & \dots & \dots \\ R(m_k/x_k, n_1/y_1) & \dots & R(m_k/x_k, n_j/y_j) \end{bmatrix}$$, where

$\mathbf{R}_{m\times n} = (R_{ij})$, where the entries of the matrix $R_{ij}, i = 1,2,\cdots, m \text{ and } j = 1,2,\cdots,n$ are defined as

$$\mathbf{R} = (R_{ij}) = \begin{cases} m_i n_j & if \left(\dfrac{m_i}{x_i}, \dfrac{n_j}{y_j}\right) \in R \\ 0 & otherwise \end{cases}$$

Example 3.1 Let $M_1 = \{2/x, 2/y\}$ and $M_2 = \{2/a, 3/b\}$, and let the mset relation R be defined on $M_1 \times M_2$ by $R = \{(2/x, 2/b)/4, (2/y, 3/b)/6, (1/y, 1/a)/1\}$, then the matrix representation of this relation is given by

M_2 M_1	$1/a$	$2/a$	$1/b$	$2/b$	$3/b$

$$\mathbf{R} = \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

We now define the compositions of relations on multisets. Suppose we have three msets M_1, M_2 and M_3. Let $R \subseteq M_1 \times M_2$ and $S \subseteq M_2 \times M_3$, then we will define a new relation known as the $max - min$ composition of the multiset relations R and S as follows.

Definition 3.2.[The $max - min$ compositions of relations on msets]

Let $x \in^m M_1,\ y \in^n M_2,\ z \in^l M_3$, then the $max - min$ composition of the mset relations R and S is defined as $(R \circ S)(x,z) = \vee_{y \in Y} (x,y) \wedge (y,z),\ (x,y) \in R, (y,z) \in S$ and is written in the matrix form as
$\mathbf{R} \circ \mathbf{S} = \left(max\{min\{r_{ik}, s_{kj}\},\ k = 1,2,\cdots n\}\right),\ i = 1,2,\cdots m,\ \ j = 1,2,\cdots l$, such that

$m_1 = n_1, m_3 \leq m_1$ or $m_3 = n_1, m_1 \leq m_3$ where m_1, n_1, m_3 is the number of occurrence of the element x in the multisets M_1, M_2 and M_3 respectively

Remark 3.1 For the $max - min$ composition of the mset relations R and S to be defined, the matching condition of the matrix multiplication must be satisfied, i.e., the number of columns of R equals to the number of rows of S.

Remark 3.2 Since matrix multiplication is not commutative, it is clear that $max - min$ composition of mset relations is not commutative.

Example 3.2 Let $M_1 = \{2/x, 2/y\}, M_2 = \{2/a, 3/b\}, M_3 = \{2/c, 2/d\}$, and let the mset relations $R \subset M_1 \times M_2$ and $S \subset M_2 \times M_3$, which are given by

$$\mathbf{R} = \begin{array}{c|c} {}_{M_1}\backslash^{M_2} & 1/a \quad 2/a \quad 1/b \quad 2/b \quad 3/b \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad \mathbf{S} = \begin{array}{c|c} {}_{M_2}\backslash^{M_3} & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} & \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

then the $max - min$ composition of R and S is

$$\mathbf{R} \circ \mathbf{S} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

or

$$\mathbf{R \circ S} = \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline 1/x & & & & \\ 2/x & & & & \\ 1/y & & & & \\ 2/y & & & & \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix},$$

which shows that

$(R \circ S) = \{(2/x, 2/c)/4, (2/x, 1/d)/2, (2/y, 2/d)/4\}$

In the following, we shall consider some properties of $max - min$ composition on multisets. To do that, suppose we have the msets M_1, M_2, M_3 and M_4 and let $R \subseteq M_1 \times M_2$, $S \subseteq M_2 \times M_3$ and $Q \subseteq M_3 \times M_4$, then we have the following properties.

Property 3.1 The $max - min$ composition is associative, if the matching condition is assured, i.e.,

$$(R \circ S) \circ Q = R \circ (S \circ Q)$$

Proof For every $(x, y) \in R, (y, z) \in S, (z, u) \in Q$, we have

$$(R \circ S) \circ Q = \vee\left(\vee\left((x,y) \wedge (y,z)\right)\right) \wedge (z,u)$$

$$= \vee\left[\vee_{y,z}\left((x,y) \wedge (y,z) \wedge (z,u)\right)\right]$$

$$= \vee\left[(x,y) \wedge \left(\vee\left((y,z) \wedge (z,u)\right)\right)\right]$$

$$= R \circ (S \circ Q)$$

Example 3.3

Let $M_1 = \{2/x, 2/y\}$, $M_2 = \{2/a, 3/b\}$, $M_3 = \{2/c, 2/d\}$, $M_4 = \{2/e, 3/f\}$,

and

$$\mathbf{R} = \begin{array}{c|ccccc} M_1 \backslash M_2 & 1/a & 2/a & 1/b & 2/b & 3/b \\ \hline 1/x & & & & & \\ 2/x & & & & & \\ 1/y & & & & & \\ 2/y & & & & & \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{S} = \begin{array}{c|cccc} M_2 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline 1/a & & & & \\ 2/a & & & & \\ 1/b & & & & \\ 2/b & & & & \\ 3/b & & & & \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$M_3 \backslash M_4$	$1/e$	$2/e$	$1/f$	$2/f$	$3/f$

$$\mathbf{Q} = \begin{array}{c} 1/c \\ 2/c \\ 1/d \\ 2/d \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

then

$$R \circ S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} \end{array},$$

$$(R \circ S) \circ Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_4 & \begin{array}{ccccc} 1/e & 2/e & 1/f & 2/f & 3/f \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array},$$

and

$$S \circ Q = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_2 \backslash M_4 & \begin{array}{ccccc} 1/e & 2/e & 1/f & 2/f & 3/f \end{array} \\ \hline \begin{array}{c} 1/a \\ 1/a \\ 1/b \\ 2/b \\ 3/b \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$R \circ (S \circ Q) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_4 & \begin{matrix} 1/e & 2/e & 1/f & 2/f & 3/f \end{matrix} \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix}, \end{array}$$

i.e.

$(R \circ S) \circ Q = R \circ (S \circ Q) = \{(1/y\,, 1/e)/1\,, (2/y\,, 1/f)/2\,\}$

Property 3.2 Let $R_1, R_2 \subseteq M_1 \times M_2$, $R_1 \leq R_2$ and $Q \subseteq M_2 \times M_3$, then

$$R_1 \circ Q \leq R_2 \circ Q$$

Proof From the definition of the $max - min$ composition on multisets we have

$R_{1\,\circ}\, Q = \vee_{y \in Y} (x, y) \wedge (y, z),\ \ \forall (x, y) \in R_1,\ \ (y, z) \in Q$

Since $R_1 \leq R_2$ by assumption, this implies $(x, y) \in R_2$. Thus we can write`

$R_{1\,\circ}\, Q \leq \vee_{y \in Y} (x, y) \wedge (y, z), \forall (x, y) \in R_2,\ \ (y, z) \in Q,$

From which we conclude that

$R_{1\,\circ}\, Q \leq R_2 \circ Q$

Example 3.4

Let $M_1 = \{2/x\,, 2/y\}$, $M_2 = \{2/a\,, 3/b\}$, $M_3 = \{2/c\,, 2/d\}$,

$$\mathbf{R_1} = \begin{array}{c|c} M_1 \backslash M_2 & \begin{matrix} 1/a & 2/a & 1/b & 2/b & 3/b \end{matrix} \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \end{array} \qquad \mathbf{R_2} = \begin{array}{c|c} M_1 \backslash M_2 & \begin{matrix} 1/a & 2/a & 1/b & 2/b & 3/b \end{matrix} \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix}, \end{array}$$

$$\begin{array}{c|c} M_2 \backslash M_3 & \begin{matrix} 1/c & 2/c & 1/d & 2/d \end{matrix} \end{array}$$

$$\mathbf{Q} = \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

then

$$R_1 \circ Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \multicolumn{4}{c}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \end{array},$$

$$R_2 \circ Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \multicolumn{4}{c}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix}} \end{array}$$

We see that

$R_1 \circ Q = \{(1/y, 1/c)/1, (2/x, 1/d)/2\},$

$R_2 \circ Q = \{(1/y, 1/c)/1, (2/x, 1/d)/2, (2/y, 2/c)/4\},$

and this show that

$$R_1 \leq R_2 \Rightarrow R_1 \circ Q \leq R_2 \circ Q \ \forall all\ msets\ R_1, R_2, Q$$

Property 3.3. For any $R, S \in M_1 \times M_2$ and $Q \in M_2 \times M_3$, then we have

(i) $(R \vee S) \circ Q = (R \circ Q) \vee (S \circ Q)$

(ii) $(R \wedge S) \circ Q \leq (R \circ Q) \wedge (S \circ Q)$

Proof. **(i)** For every $(x, y) \in R, (y, z) \in S, (z, u) \in Q$, we have

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$$(R \vee S) \circ Q = \vee_{y \in Y} (R \vee S) \wedge Q$$

$$= \vee_{y \in Y} \big((x, y) \wedge (y, z)\big) \vee \big((x, y) \wedge (y, z)\big)$$

$$\leq \vee_{y \in Y} \big((x, y) \wedge (y, z)\big) \vee \vee_{y \in Y} \big((x, y) \wedge (y, z)\big)$$

Then $(R \vee S) \circ Q \leq (R \circ Q) \vee (S \circ Q)$

Since $(R \circ Q) \leq (R \vee S) \circ Q$

And $(S \circ Q) \leq (R \vee S) \circ Q$

$(R \circ Q) \vee (S \circ Q) \leq (R \vee S) \circ Q$

$(R \vee S) \circ Q = (R \circ Q) \vee (S \circ Q)$

(ii)

$$(R \wedge S) \circ Q(x, z) = \vee_{y \in Y} (R \wedge S)(x, y) \wedge Q(y, z)$$

$$= \vee_{y \in Y} \big(R(x, y) \wedge Q(y, z)\big) \wedge \big(S(x, y) \wedge Q(y, z)\big)$$

$$\leq \vee_{y \in Y} \big(R(x, y) \wedge Q(y, z)\big) \wedge \vee_{y \in Y} \big(S(x, y) \wedge Q(y, z)\big)$$

$$(R \wedge S) \circ Q \leq (R \circ Q) \wedge (S \circ Q)$$

Example 3.5

let $M_1 = \{2/x, 2/y\}$, $M_2 = \{2/a, 3/b\}$, $M_3 = \{2/c, 2/d\}$,

And

$$\mathbf{R} = \begin{array}{c|c} M_1 \backslash M_2 & \quad 1/a \quad 2/a \quad 1/b \quad 2/b \quad 3/b \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad \mathbf{S} = \begin{array}{c|c} M_1 \backslash M_2 & \quad 1/a \quad 2/a \quad 1/b \quad 2/b \quad 3/b \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array},$$

$$\begin{array}{ccccc} M_3 & 1/c & 2/c & 1/d & 2/d \\ M_2 & & & & \end{array}$$

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$$\mathbf{Q} = \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

then

$R \vee S = Max\{\mathbf{R}, \mathbf{S}\} = max\{r_{ij}, s_{ij}\}, \quad i = 1,2, \cdots m, \quad j = 1,2, \cdots n$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 \end{bmatrix},$$

$R \wedge S = Min\{\mathbf{R}, \mathbf{S}\} = min\{r_{ij}, s_{ij}\}, \quad i = 1,2, \cdots m, \quad j = 1,2, \cdots n$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

And

$$\mathrm{R} \circ \mathrm{Q} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \multicolumn{4}{c}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix}} \end{array},$$

$$\mathrm{S} \circ \mathrm{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \multicolumn{4}{c}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}} \end{array},$$

=

Using the above matrices, we get

$$(R \vee S) \circ Q = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_3 \backslash M_1 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \end{bmatrix} \end{array}, \qquad (1)$$

And

$$(R \circ Q) \vee (S \circ Q) = \begin{array}{c|c} M_3 \backslash M_1 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 0 & 2 & 0 \end{bmatrix} \end{array} \qquad (2)$$

Moreover,

$$(R \wedge S) \circ Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_3 \backslash M_1 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad (3)$$

$$(R \circ Q) \wedge (S \circ Q) = \begin{array}{c|c} M_3 \backslash M_1 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (4)$$

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From (1) and (2) we see that

$$(R \vee S) \circ Q = (R \circ Q) \vee (S \circ Q) = \{(1/x, 1/c)/1, (2/x, 1/c)/2, (1/y, 1/c)/1, (2/y, 1/c)/2, (2/y, 1/d)/2, (2/x, 1/d)/2\}$$

$$(R \wedge S) \circ Q = (R \circ Q) \wedge (S \circ Q) = \{(1/y, 1/c)/1\}$$

4. New definitions on $min - max$ composition

In this section, we introduce the second type of definition of composition on multiset by consider some conditions on the count of each multisets to satisfied the multi relation condition on multiset. Suppose we have three msets M_1, M_2 and M_3. Let $R \subseteq M_1 \times M_2$ and $S \subseteq M_2 \times M_3$, then we will define a new relation known as the $min - max$ composition of the multiset relations R and S as follows.

Definition 4.1 [The $min - max$ compositions of relations on msets]

Let $x \in^m M_1$, $y \in^n M_2$, $z \in^l M_3$, then the $min - max$ composition of the mset relations R and S is defined as $R \bullet S(x,z) = \wedge_{y \in Y} (x,y) \vee (y,z)$, $(x,y) \in R, (y,z) \in S$ Such that the number of occurrence of n_1 equal one and the number of occurrence of m_1 or m_3 equal one or all the numbers of occurrence of the elements are equal where m_1, n_1, m_3 is the number of occurrence of the element x in the multiset M_1, M_2 and M_3 respectively and

$$\mu(x,y) = \begin{cases} Min(\mu_x, \mu_y) & if\ \mu(x) \neq 0, \mu(y) \neq 0 \\ 0 & otherwise \end{cases}$$

Where μ_x, μ_y the count of the element in multisets R and S and $\mu(x,y)$ the count of element in $R \bullet S$.

Example 4.1 Let $M_1 = \{2/x, 2/y\}, M_2 = \{2/a, 3/b\}, M_3 = \{2/c, 2/d\}$, and let the mset relations $R \subset M_1 \times M_2$ and $S \subset M_2 \times M_3$ such that,

$$\mathbf{R} = \begin{array}{c|c} M_1 \backslash M_2 & \begin{array}{ccccc} 1/a & 2/a & 1/b & 2/b & 3/b \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}, \quad \mathbf{S} = \begin{array}{c|c} M_2 \backslash M_3 & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \hline \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} & \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \end{array}$$

Then the $min - max$ composition of R and S is

$$R \bullet S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix},$$

$$R \bullet S = \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline 1/x & & & & \\ 2/x & & & & \\ 1/y & & & & \\ 2/y & & & & \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

which shows that

$R \bullet S = \{(2/x, 1/c)/2, (1/y, 2/d)/2\}$

Example 4.2: Let $M_1 = \{2/x, 2/y\}, M_2 = \{2/a, 3/b\}, M_3 = \{2/c, 2/d\}$ and let the mset

relations $R \subset M_1 \times M_2$ and $S \subset M_2 \times M_3$ such that,

$$\mathbf{R} = \begin{array}{c|ccccc} M_1 \backslash M_2 & 1/a & 2/a & 1/b & 2/b & 3/b \\ \hline 1/x & & & & & \\ 2/x & & & & & \\ 1/y & & & & & \\ 2/y & & & & & \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix}, \qquad \mathbf{S} = \begin{array}{c|cccc} M_2 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline 1/a & & & & \\ 2/a & & & & \\ 1/b & & & & \\ 2/b & & & & \\ 3/b & & & & \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then the $min - max$ composition of R and S is

$$R \bullet S = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R \bullet S = \begin{array}{c|cccc} M_1 \backslash M_3 & 1/c & 2/c & 1/d & 2/d \\ \hline 1/x & & & & \\ 2/x & & & & \\ 1/y & & & & \\ 2/y & & & & \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix},$$

which shows that

$R \bullet S = \{(2/x, 2/c)/4, (2/y, 2/d)/4\}$

In the following, we shall consider some properties of $min - max$ composition on multisets. To do that, suppose we have the msets M_1, M_2, M_3 and M_4 and let $R \subseteq M_1 \times M_2$, $S \subseteq M_2 \times M_3$ and $T \subseteq M_3 \times M_4$, then we have the following properties.

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Property 4.1 The $min - max$ composition is associative, if the matching condition is assured, i.e.
$(R \bullet S) \bullet T = R \bullet (S \bullet T)$

Proof

$$(R \bullet S) \bullet T = \wedge_{z \in Z} \left[\wedge_{y \in Y} \big(R(x,y) \vee S(y,z) \big) \right] \vee_z T(z,u)$$

$$= \wedge_{y \in Y} [R(x,y) \vee (\wedge_{z \in Z} \big(S(y,z) \vee T(z,u) \big)]$$

$$= R \bullet (S \bullet T)$$

Example 4.3

Let $M_1 = \{2/x, 2/y\}, M_2 = \{2/a, 3/b\}, M_3 = \{2/c, 2/d\}, M_4 = \{2/e, 3/f\}$, and

$$\mathbf{R} = \begin{array}{c|c} M_1 \backslash M_2 & \begin{matrix} 1/a & 2/a & 1/b & 2/b & 3/b \end{matrix} \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}, \quad \mathbf{S} = \begin{array}{c|c} M_2 \backslash M_3 & \begin{matrix} 1/c & 2/c & 1/d & 2/d \end{matrix} \\ \hline \begin{matrix} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$\mathbf{T} = \begin{array}{c|c} M_3 \backslash M_4 & \begin{matrix} 1/e & 2/e & 1/f & 2/f & 3/f \end{matrix} \\ \hline \begin{matrix} 1/c \\ 2/c \\ 1/d \\ 2/d \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array},$$

then

$$R \bullet S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & \begin{matrix} 1/c & 2/c & 1/d & 2/d \end{matrix} \\ \hline \begin{matrix} 1/x \\ 2/x \\ 1/y \\ 2/y \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \end{array},$$

$$(R \bullet S) \bullet T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_4 & \begin{array}{ccccc} 1/e & 2/e & 1/f & 2/f & 3/f \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array},$$

and

$$S \bullet T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_2 \backslash M_4 & \begin{array}{ccccc} 1/e & 2/e & 1/f & 2/f & 3/f \end{array} \\ \hline \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$R \bullet (S \bullet T) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_4 & \begin{array}{ccccc} 1/e & 2/e & 1/f & 2/f & 3/f \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array},$$

i.e.

$R \bullet (S \bullet T) = (R \bullet S) \bullet T = \{(2/x, 2/e)/4, (2/y, 2/f)/4\}$

Property 4.2: Let $R_1, R_2 \subseteq M_1 \times M_2$, $R_1 \leq R_2$ and $T \subseteq M_2 \times M_3$, then

$R_1 \bullet T \leq R_2 \bullet T$

Proof:

$R_1 \bullet T(x, z) = \wedge_{y \in Y} R_1(x, y) \vee T(y, z)$

$\leq \wedge_{y\in Y} R_2(x,y) \vee T(y,z) = R_2 \bullet T(x,z)$

Example 4.4: Let $M_1 = \{2/x\,, 2/y\}, M_2 = \{2/a\,, 3/b\}, M_3 = \{2/c\,, 2/d\}$ and

$$\mathbf{R_1} = \begin{array}{cc} {}_{M_1}\backslash^{M_2} & \begin{array}{ccccc} 1/a & 2/a & 1/b & 2/b & 3/b \end{array} \\ \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}, \quad \mathbf{R_2} = \begin{array}{cc} {}_{M_1}\backslash^{M_2} & \begin{array}{ccccc} 1/a & 2/a & 1/b & 2/b & 3/b \end{array} \\ \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$\mathbf{T} = \begin{array}{cc} {}_{M_2}\backslash^{M_3} & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

then

$$R_1 \bullet T = \begin{bmatrix} 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} {}_{M_1}\backslash^{M_3} & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

$$R_2 \bullet T = \begin{bmatrix} 0 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{cc} {}_{M_1}\backslash^{M_3} & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We see that

$R_1 \bullet T = \{(1/y\,, 2/c)/2\},$

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$R_2 \bullet T = \{(1/y\,,2/c)/2\,,(2/x\,,1/c)/2\}$

and this show that
$R_1 \le R_2 \Rightarrow R_1 \bullet T \le R_2 \bullet T\ \forall all\ msets\ R_1, R_2, Q$

Property 4.3 For any $R, S \in M_1 \times M_2$ and $T \in M_2 \times M_3$, then we have

(i) $(R \wedge S) \bullet T = (R \bullet T) \wedge (S \bullet T)$
(ii) $(R \vee S) \bullet T \ge (R \bullet T) \vee (S \bullet T)$

Proof

(i)$(R \wedge S) \bullet T(x,z) = \wedge_{y \in Y} (R \wedge S)(x,y) \vee T(y,z)$

$= \wedge_{y \in Y} \big(R(x,y) \vee T(y,z)\big) \wedge \big(S(x,y) \vee T(y,z)\big)$

$\ge \wedge_{y \in Y} \big(R(x,y) \vee T(y,z)\big) \wedge \wedge_{y \in Y} \big(S(x,y) \vee T(y,z)\big)$

Then $(R \wedge S) \bullet T \ge (R \bullet T) \wedge (S \bullet T)$

Since $(R \bullet T) \ge (R \wedge S) \bullet T$

And $(S \bullet T) \ge (R \wedge S) \bullet T$

$(R \bullet T) \wedge (S \bullet T) \ge (R \wedge S) \bullet T$

$(R \wedge S) \bullet T = (R \bullet T) \wedge (S \bullet T)$

(ii)$(R \vee S) \bullet T(x,z) = \wedge_{y \in Y} (R \vee S)(x,y) \vee T(y,z)$

$= \wedge_{y \in Y} \big(R(x,y) \vee T(y,z)\big) \vee \big(S(x,y) \vee T(y,z)\big)$

$\ge \wedge_{y \in Y} \big(R(x,y) \vee T(y,z)\big) \vee \wedge_{y \in Y} \big(S(x,y) \vee T(y,z)\big)$

Example 4.5 Let $M_1 = \{2/x\,,2/y\},\ M_2 = \{2/a\,,3/b\}, M_3 = \{2/c\,,2/d\}$ and

$$\mathbf{R} = \begin{array}{c} M_1 \backslash M_2 \\ 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} \begin{array}{c} \begin{array}{ccccc} 1/a & 2/a & 1/b & 2/b & 3/b \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}, \quad \mathbf{S} = \begin{array}{c} M_1 \backslash M_2 \\ 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} \begin{array}{c} \begin{array}{ccccc} 1/a & 2/a & 1/b & 2/b & 3/b \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array},$$

M_3 / M_2	$1/c$	$2/c$	$1/d$	$2/d$

$$\mathbf{T} = \begin{array}{c} 1/a \\ 2/a \\ 1/b \\ 2/b \\ 3/b \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

then

$$R \vee S = Max\{\mathbf{R}, \mathbf{S}\} = max\{r_{ij}, s_{ij}\}, \quad i = 1,2, \cdots m, \quad j = 1,2, \cdots n$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 6 \end{bmatrix},$$

$$R \wedge S = Min\{\mathbf{R}, \mathbf{S}\} = min\{r_{ij}, s_{ij}\}, \quad i = 1,2, \cdots m, \quad j = 1,2, \cdots n$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

And

$$\mathrm{R} \bullet \mathrm{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array},$$

$$\mathrm{S} \bullet \mathrm{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & \begin{array}{cccc} 1/c & 2/c & 1/d & 2/d \end{array} \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array},$$

Using the above matrices, we get

$$(R \vee S)\bullet Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}, \qquad (1)$$

and

$$(R\bullet Q)\vee(S\bullet Q) = \begin{array}{c|c} M_1 \backslash M_3 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array} \qquad (2)$$

Moreover,

$$(R \wedge S)\bullet Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{array}{c|c} M_1 \backslash M_3 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad (3)$$

$$(R\bullet Q)\wedge(S\bullet Q) = \begin{array}{c|c} M_1 \backslash M_3 & 1/c \quad 2/c \quad 1/d \quad 2/d \\ \hline \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \qquad (4)$$

From (1) and (2) we see that

$$(R \vee S)\bullet Q = (R\bullet Q)\vee(S\bullet Q)$$
$$= \{(1/x\,,1/c)/1\,,(1/y\,,1/d)/1\,,(2/y\,,1/d)/2\,\}$$
$$(R \wedge S)\bullet Q = (R\bullet Q)\wedge(S\bullet Q) = \{(1/x\,,1/c)/1\,\}$$

5. New results on multiset classifications

In this section, we define after and for multiset and use the definition of composition on multiset to obtain new result on multiset relation.

Definition 5.1 Let R be an mset relation on M the after set of $x \in M$ is defined as
$(m/x)R = \{n/y : \exists some\ k\ with\ (k/x)R(n/y)\}$
And the for set of $x \in M$ is defined as
$R(n/y) = \{m/x : \exists some\ q\ with\ (m/x)R(q/y)\}$,
where $m, n, k, q \in \{0,1,2,3, \dots \dots \dots ..\}$.

Proposition 5.1 Let R and S be multiset relations and $R \circ S$is the max-min composition of the multiset relations R and S then the following properties are defined:

(i) $X \subseteq Y$ then $XR \subseteq YR$
(ii) $X \subseteq Y$ then $X(R \circ S) \subseteq Y(R \circ S)$
(iii) $X(R \circ S) \cap Y(R \circ S) = (X \cap Y)R \circ S$
(iv) $X(R \circ S) \cup Y(R \circ S) = (X \cup Y)R \circ S$
(v) $X(R \circ S) \oplus Y(R \circ S) \supseteq (X \oplus Y)R \circ S$
(vi) $X(R \circ S) \ominus Y(R \circ S) \subseteq (X \ominus Y)R \circ S$

Example 5.1 Let $M = \{2/x, 2/y, 2/z\}$ and
$R = \{(2/x, 2/y)/4, (1/x, 2/z)/2, (1/y, 1/x)/1, (1/z, 1/y)/1\}$
$S = \{(2/x, 1/y)/2, (2/z, 2/x)/4, (1/x, 1/z)/1, (1/y, 1/z)/1\}$
$(1/x)R = (2/x)R = \{2/x, 2/z\}$
$(1/y)R = \{1/x\}$
$(1/z)R = \{1/y\}$
After set for all value of M =$\{(2/x, 2/z), (1/x), (1/y)\}$

$$R \circ S = \begin{array}{c} 1/x \\ 2/x \\ 1/y \\ 2/y \\ 1/z \\ 2/z \end{array} \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R \circ S = \{(1/x, 2/x)/2, (2/x, 1/y)/2, (1/y, 1/z)/1, (1/z, 1/z)/1\}$
After set for all value of $R \circ S$=$\{(2/x, 1/y), (1/z)\}$
Let $X = \{2/x\}$, Y=$\{2/x, 1/z\}$ then

(i)$(2/x)R = (2/x, 2/z)$ $(2/x, 1/z)R = (2/x, 2/z, 1/y)$
(ii)$(2/x)R \circ S = (2/x, 1/y)$ $(2/x, 1/z)R \circ S = (2/x, 1/y, 1/z)$

Let $X = (2/x, 1/y), Y = (1/y, 1/z)$, then

(iii) $X(R \circ S) \cap Y(R \circ S) = (1/z)$,$(X \cap Y)R \circ S = (1/z)$
(iv)$X(R \circ S) \cup Y(R \circ S) = (2/x, 1/y, 1/z), (X \cup Y)R \circ S = (2/x, 1/y, 1/z)$
(v)$X(R \circ S) \oplus Y(R \circ S) = (2/x, 1/y, 2/z), (X \oplus Y)(R \circ S) = (2/x, 1/y, 1/z)$

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(vi)$X(R\circ S)\ominus Y(R\circ S)=(2/x\,,1/y),(X\ominus Y)(R\circ S)=(2/x\,,1/y)$ and if , X=$(2/x\,,1/y),Y=(1/z)$then $X(R\circ S)\ominus Y(R\circ S)=(2/x\,,1/y)$, $(X\ominus Y)R\circ S=(2/x\,,1/y,1/z)$

Proposition 5.2 Let R and S be multiset relations and $R\circ S$is the max-min composition of the multiset relations R and S then the following properties are defined:

(i)$X\subseteq Y$ then $RX\subseteq RY$
(ii)$X\subseteq Y$ then $(R\circ S)X\subseteq(R\circ S)Y$
(iii)$(R\circ S)X\cap(R\circ S)Y=R\circ S(X\cap Y)$
(iv)$(R\circ S)X\cup(R\circ S)Y=R\circ S(X\cup Y)$
(v)$(R\circ S)X\oplus(R\circ S)Y\supseteq R\circ S(X\oplus Y)$
(vi)$(R\circ S)X\ominus(R\circ S)Y\subseteq R\circ S(X\ominus Y)$

Proposition 5.3 Let R and S be multiset relations and $R\bullet S$is the min-max composition of the multiset relations R and S then the following properties are defined:

(i)$X\subseteq Y$ then $RX\subseteq RY$
(ii)$X\subseteq Y$ then $(R\bullet S)X\subseteq(R\bullet S)Y$
(iii)$(R\bullet S)X\cap(R\bullet S)Y=R\bullet S(X\cap Y)$
(iv)$(R\bullet S)X\cup(R\bullet S)Y=R\bullet S(X\cup Y)$
(v)$(R\bullet S)X\oplus(R\bullet S)Y\supseteq R\bullet S(X\oplus Y)$
(vi)$(R\bullet S)X\ominus(R\bullet S)Y\subseteq R\bullet S(X\ominus Y)$

Example 5.2 Let $M=\{2/x\,,2/y\,,2/z\}$ and

$R=\{(2/x\,,1/y)/2,(1/x\,,1/z)/1\,,(1/y\,,1/x)/1,(2/z\,,2/x)/4\}$

$S=\{(1/y\,,1/x)/1,(1/z\,,1/z)/1\,,(1/x\,,2/z)/2\}$

$(1/x)R=(2/x)R=\{1/y,1/z\}$

$(1/y)R=\{1/x\}$

$(2/z)R=\{2/x\}$

After set for all value of M =$\{(1/y\,,1/z),(1/x),(2/x)\}$

$$R\bullet S=\begin{array}{c}1/x\\2/x\\1/y\\2/y\\1/z\\2/z\end{array}\begin{bmatrix}0&0&0&0&1&0\\2&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&0\end{bmatrix}$$

$R \bullet S = \{(2/x, 1/x)/2, (1/x, 1/z)/1, (1/y, 2/z)/2,\}$

After set for all values of $R \bullet S$=$\{(1/x, 1/z), (2/z)\}$

Let $X = \{1/x\}$,$Y = \{1/x, 1/y\}$, then

(i) $(1/x)R = (1/y, 1/z)$ $(1/x, 1/y)R = (1/y, 1/z, 1/x)$
(ii) $(1/x)R \bullet S = (1/x, 1/z)$ $(1/x, 1/y)R \bullet S = (1/x, 1/z, 2/z)$

Let $X = (1/x, 1/y)$, $Y = (1/x, 1/z)$ then

(iii) $X(R \bullet S) \cap Y(R \bullet S) = (1/x, 1/z)$,$(X \cap Y)R \bullet S = (1/x, 1/z)$
(iv) $X(R \bullet S) \cup Y(R \bullet S) = (1/x, 1/z, 2/z)$, $(X \cup Y)R \bullet S = (1/x, 1/z, 1/z)$
(v) Let
$X = (1/z, 1/y), Y = (2/x, 1/z)$ *then* $X(R \bullet S) \oplus Y(R \bullet S) = (1/x, 3/z), (X \oplus Y)(R \bullet S) = (1/x, 1/z, 2/z)$

6. Conclusion

The suggested notions for matrix representations of multiset relations can open the way for simple methods in constructing approximation spaces on multiset information systems, and the composition operation can help in successive effect of operations. In the future work we can apply these definitions on rough and fuzzy multiset theory. Also, we can apply this work in some application on graph theory and decision making on multisets.

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