On Some Relations of fuzzy z-open set in fuzzy topology space on fuzzy set

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Abstract

The aim of this paper is to introduce and study a fuzzy Z-open set (resp .fuzzy b-open ,fuzzy β –open, fuzzy β^* -open, fuzzy e-open)sets in fuzzy topology space on fuzzy set and devote to study some properties and the relation between them.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadah ,chang introduced the definition of fuzzy topology space, Chakrabarty ,M.K. and Ahsanullal,T,M.G. " Fuzzy topological space on fuzzy sets And Tolerance topology" Fuzzy sets and system 45,103108(1992).Chaudhuri , A .k. and Das ,P. ,"Some Results on Fuzzy Topological On Fuzzy Sets" , Fuzzy sets and systems 56 , 331-336 , (1993), EI.Magharabi, MohammedA.AL- juhani New Types Of Fuction By M-open Sets Of Taibah university For Science Joumal In (2013).In this paper , we introduce some types of fuzzy open sets in fuzzy topological space on fuzzy set and study some relations and given some counter examples.

2. Fuzzy Topological Space On Fuzzy Set

2.1 Definition [L.A.ZADEH]

Let X be a non-empty set , a fuzzy set $\tilde{A}$ in X is characterized by a function $\mu_{\tilde{A}}$: X→ I , where I =[0,1] which is written $\tilde{A}$={(x, $\mu_{\tilde{A}}$ (x)) : x ∈ X, 0 ≤ $\mu_{\tilde{A}}$ (x)≤ 1},The collection of all fuzzy sets in X will be denoted by I^X, that is I^X={$\tilde{A}$:$\tilde{A}$ is fuzzy sets in X} where $\mu_{\tilde{A}}$ is called the membership function.

2.2 Definition [Chakrabarty ,M.K. and Ahsanullal,T,M.G.]

A collection $\tilde{T}$of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq$ P($\tilde{A}$) is said to be fuzzy topology on $\tilde{A}$, if it satisfied the following conditions :

1) $\tilde{A}, \tilde{\emptyset} \in \tilde{T}$
2) If $\tilde{B}$,$\tilde{C}$ $\in \tilde{T}$, then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3) If $\tilde{B}_\alpha \in \tilde{T}$, then $\cup_\alpha \tilde{B}_\alpha \in \tilde{T}$,α ∈ Λ

($\tilde{A}, \tilde{T}$) is said to be fuzzy topological space and every member of $\tilde{T}$ is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set .

2.3Definition [Chaudhuri , A .k. and Das ,P.]

Let $\tilde{B}$ be a fuzzy set in a fuzzy topological space($\tilde{A}, \tilde{T}$) then :

• The interior of B is denoted by Int ($\tilde{B}$) and defined by

Int($\tilde{B}$)= max {$\mu_{\tilde{G}}$ (x) : $\tilde{G}$ is a fuzzy open set in $\tilde{A}$, $\mu_{\tilde{G}}$(x) ≤ $\mu_{\tilde{B}}$(x) }

• The closure of B is denoted by CL($\tilde{B}$) and defined by

CL ($\tilde{B}$)= min { $\mu_{\tilde{F}}$: $\tilde{F}$ is a fuzzy closed set in $\tilde{A}$, $\mu_{\tilde{B}}$(x) $\leq \mu_{\tilde{F}}$(x) }

2.4 Definition [Shymaa Abd Alhassan A]

A fuzzy set $\tilde{B}$in a fuzzy topological space ($\tilde{A}$,$\tilde{T}$) is said to be

• $Fuzzy\ \delta - open\ set\ if\ \mu_{(Int(cl(\tilde{B})))}$ (x) $\leq$ $\mu_{\tilde{B}}$ (x) .

• Fuzzy $\delta - closed\ set\ if\ \mu_{\tilde{B}}$ (x) $\leq \mu_{(cl(Int(\tilde{B})))}$ (x) .

2.5 Definition [EI.Magharabi, MohammedA.AL- juhani**]**

A fuzzy set $\tilde{D}$ in a topological space ($\tilde{A}$,$\tilde{T}$) is said to be

• Fuzzy Z-open set if $\mu_{\tilde{D}}$ $(x) \leq$ max {$\mu_{(cl(Int_{\delta}(\tilde{D}))}$ (x) , $\mu_{(Int(cl(\tilde{D})))}$ (x)}.

•Fuzzy Z-closed set if min {$\mu_{(cl(Int_{\delta}(\tilde{D}))}$ (x) , $\mu_{(cl(int(\tilde{D})))}$ (x)} $\leq$ $\mu_{\tilde{D}}$ (x) .

2.6 Definition:

Let $\tilde{D}$ be a fuzzy set in a fuzzy topological space($\tilde{A}$, $\tilde{T}$) then :

• The Z- interior of $\tilde{D}$ is denoted by Z Int $(\tilde{D})$ and defined by

ZInt($\tilde{D}$) = max {$\mu_{\tilde{G}}$ (x): $\tilde{G}$ is a fuzzy z- open set in $\tilde{A}$, $\mu_{\tilde{G}}$(x) $\leq \mu_{\tilde{D}}$(x)}

• The Z- closure of $\tilde{D}$is denoted by Z CL$(\tilde{D})$and defined by

Zcl ($\tilde{D}$) = min {$\mu_{\tilde{S}}$ (x): $\tilde{S}$is a fuzzy Z-closed set in Ã , $\mu_{\tilde{D}}$ (x)$\leq \mu_{\tilde{S}}$ (x) }

Definition 2.7[Chakrabarty ,M.K. and Ahsanullal,T,M.G**]**

Let $\tilde{B}$, $\tilde{C}$ be a fuzzy set in a fuzzy topological space ($\tilde{A}$, $\tilde{T}$) then :

• A fuzzy point x_r is said to be quasi coincident with the fuzzy set $\tilde{B}$ if there exist x ∈ X such that μ_{x_r}(x)+ $\mu_{\tilde{B}}$(x) > $\mu_{\tilde{A}}$ (x) and denoted by $x_r\ q\ \tilde{B}$, if μ_{x_r}(x) + $\mu_{\tilde{B}}$(x) $\leq \mu_{\tilde{A}}$ (x) $\forall x \in X$ then x_r is not quasi coincident with a fuzzy set $\tilde{B}$ and is denoted by $x_r\ \tilde{q}\ \tilde{B}$.

• A fuzzy set $\tilde{B}$ is said to be quasi coincident (overlap) with a fuzzy set $\tilde{C}$ if there exist x ∈ Xsuch that $\mu_{\tilde{B}}$ (x) +$\mu_{\tilde{C}}$(x) > $\mu_{\tilde{A}}$(x) and denoted by $\tilde{B}\ q\ \tilde{C}$, if $\mu_{\tilde{C}}$ (x) + $\mu_{\tilde{B}}$(x) $\leq \mu_{\tilde{A}}$ (x) $\forall x \in X$ then $\tilde{B}$ is not quasi coincident with a fuzzy set $\tilde{C}$ and is denoted by $\tilde{B}\ \tilde{q}\ \tilde{C}$.

2.8 Remark:

1. The intersection of two fuzzy Z-open set is not necessary a fuzzy Z-open set.

2. The union of two fuzzy Z-closed set is not necessary a fuzzy Z-closed set .

As shown by the following example :-

2.9 Example:

Let X={a , b } and $\tilde{B}$,$\tilde{C}$,$\tilde{D}$,$\tilde{E}$,$\tilde{F}$,$\tilde{G}$,$\tilde{H}$ be fuzzy subset of Ã where :

Ã ={ (a,0.6) ,(b,0.8)} , $\tilde{B}$={ (a,0.0) ,(b,0.6)}

$\tilde{C}$ = { (a,0.6) ,(b,0.0)} , $\tilde{D}$ = { (a,0.6) ,(b,0.6)}

$\tilde{E}$= { (a,0.4) ,(b,0.4)} , $\tilde{F}$ ={ (a,0.0) ,(b,0.3)}

$\tilde{G}$= { (a,0.0),(b,0.4)} , $\tilde{H}$= { (a,0.4) ,(b,0.6)}

$\tilde{k}$ = { (a,0.6),(b,0.4)} , $\tilde{M}$= { (a,0.6),(b,0.3)}

$\tilde{N}$ = { (a,0.4),(b,0.3)} ,$\tilde{W}$= { (a,0.6),(b,0.2)}

And $\tilde{T}$= { $\tilde{\emptyset}$,Ã ,$\tilde{B}$,$\tilde{C}$, $\tilde{D}$,$\tilde{E}$, $\tilde{F}$,$\tilde{G}$, $\tilde{H}$, $\tilde{k}$, $\tilde{M}$, $\tilde{N}$ } be a fuzzy topological on Ã
$\tilde{B}$ and $\tilde{W}$ are fuzzy Z-open sets but $\tilde{B}$ ∩ $\tilde{W}$ is not fuzzy Z-open set.

2.10 Proposition:

Let (Ã,$\tilde{T}$) be a fuzzy topological space and $\tilde{D}$ ∈ p(Ã),

Then $\mu_{zint(\tilde{D})}$(x) = $\mu_{(zcl(\tilde{D}^c))^c}$ (x) ∀ x ∈X

<u>**Proof :**</u>

Since $\mu_{zint(\tilde{D})}$ (x) ≤ $\mu_{\tilde{D}}$ (x)

And Zint ($\tilde{D}$) is a fuzzy Z-open set

Then $\mu_{\tilde{D}^c}$(x) ≤ $\mu_{zint(\tilde{D})^c}$(x) and $\mu_{zcl(\tilde{D})^c}(x) \leq$ $\mu_{zint(\tilde{D})^c}$(x) ,

Hence$\mu_{zint(\tilde{D})}$(x) ≤ $\mu_{(zcl(\tilde{D}^c))^c}$ (x) ……….(*)

Since $\mu_{\tilde{D}^c}$ (x) ≤ $\mu_{zcl(\tilde{D})^c}$ (x) and $zcl(\tilde{D})$is a fuzzy Z-closed set

Then$\mu_{(zcl(\tilde{D}^c))^c}$ (x) ≤ $\mu_{\tilde{D}}$(x), Hence $\mu_{(zcl(\tilde{D}^c))^c}$ (x) ≤ $\mu_{zint(\tilde{D})}$ (x)……….(**)

From (*) and (**) we get $\mu_{zint(\tilde{D})}$ (x) =$\mu_{(zcl(\tilde{D}^c))^c}$ (x).

2.11 Definition

Let $(\tilde{A},\tilde{T})$ be a fuzzy topological space $\mu_{\tilde{B}}$ be a fuzzy set of $\tilde{A}$ a fuzzy point x ∈ Ã is said to be a fuzzy Z- limit point of $\tilde{B}$ if for every fuzzy Z-open set $\tilde{G}$ we have $(\tilde{G} - x) \cap \tilde{B} = \tilde{\emptyset}$

We shall call the set of all fuzzy Z- limit point of Ã the Z-derived set of Ã and denoted by d(Ã).

2.12 Proposition

Let $(\tilde{A},\tilde{T})$ be a fuzzy topological space $\tilde{B} \subseteq \tilde{C} \subseteq \tilde{A}$ then:

1) Cl $(\tilde{B}) = \tilde{B} \cup$ d $(\tilde{B})$
2) $\tilde{B}$ is Z-closed set iff d$(\tilde{B})$
3) Cl $(\tilde{B}) \subset$ d $(\tilde{B})$

Proof:

($\Longrightarrow$) Let $x \notin \mu_{cl(\tilde{B})}$ then there exist a Z-closed set $\mu_{\tilde{F}}$ in $\mu_{\tilde{A}}$ such that $\mu_{\tilde{A}} \subseteq \mu_{\tilde{F}}$ and $x \notin \mu_{\tilde{F}}$

Hence $\mu_{\tilde{V}} = x - \mu_{\tilde{F}}$ is Z-open set such that $x \in \mu_{\tilde{V}}$ and min $\{\mu_{\tilde{V}}, \mu_{\tilde{A}}\} = \mu_{\tilde{\emptyset}}$

Therefor $x \notin \mu_{\tilde{B}}$ and $x \notin \mu_{d(\tilde{B})}$, then $x \notin max\ \{\mu_{\tilde{B}}, \mu_{d(\tilde{B})}\}$

Thus max$\{\mu_{\tilde{B}}, \mu_{cl(\tilde{B})}\} \subseteq \mu_{d(\tilde{B})}$.

($\Longleftarrow$) $x \notin max\ \{\mu_{\tilde{B}}, \mu_{d(\tilde{B})}\}$ implies that there exist a Z-open set $\mu_{\tilde{V}}$ in $\mu_{\tilde{A}}$ such that $x \in \mu_{\tilde{V}}$ and

min $\{\mu_{\tilde{V}}, \mu_{\tilde{A}}\} = \mu_{\tilde{\emptyset}}$.

Hence $\mu_{\tilde{F}} = \mu_{\tilde{A}} - \mu_{\tilde{V}}$ is Z-closed set in $\mu_{\tilde{A}}$ such that $\mu_{\tilde{A}} \subseteq \mu_{\tilde{F}}$ and $x \notin \mu_{\tilde{F}}$

Hence $x \notin \mu_{cl(\tilde{A})}$, Thus $\mu_{cl(\tilde{A})} \subseteq$ max $\{\mu_{\tilde{A}}, \mu_{cl(\tilde{A})}\}$

Therefor $\mu_{cl(\tilde{B})} = \{\mu_{\tilde{B}}, \mu_{d(\tilde{B})}\}$, For (2) and (3) the proof is easy.

2.13 Proposition

Let $(\tilde{A},\tilde{T})$ be a fuzzy topological space and $\tilde{B} \subset \tilde{A}$, then cl $(\tilde{B})$ is the smallest Z-closed set containing $\tilde{B}$

Proof :

Suppose that $\tilde{C}$ is Z-closed set such that $\mu_{\tilde{B}} \subseteq \mu_{\tilde{C}}$ .since $\mu_{cl(\tilde{B})} = \mu_{\tilde{B}} \cup \mu_{d(\tilde{B})}$ by proposition 2.11(1),

and $\mu_{\tilde{B}} \subseteq \mu_{\tilde{C}}$ then $\mu_{cl(\tilde{B})} = \mu_{\tilde{B}} \cup \mu_{d(\tilde{B})} \subseteq \mu_{\tilde{C}} \cup \mu_{cl(\tilde{C})} \subseteq \mu_{\tilde{C}}$. Thus $\mu_{cl(\tilde{B})} \subseteq \mu_{\tilde{C}}$.

Therefor $\mu_{cl(\tilde{B})}$ is the smallest Z-closed set containing $\tilde{B}$.

2.14 Proposition

Let (Ã,$\tilde{T}$) be a fuzzy topological space , if $\tilde{G}$ be a fuzzy open set in Ã and $\tilde{B}$ is a fuzzy Z-open sets in Ã , then $\tilde{G} \cap \tilde{B}$ is a fuzzy Z-open set

Proof :

Since $\mu_{\tilde{G}}$ (x) = $\mu_{int(\tilde{G})}$ (x) and $\mu_{(\tilde{B})}$ (X)= $\mu_{zint(\tilde{B})}$ (X) then

Min {$\mu_{int(\tilde{G})}$ (x) ,$\mu_{zint(\tilde{B})}$ (x) } ≤ μ zint (min { $\mu_{\tilde{G}}$ (x), $\mu_{\tilde{B}}$ (x)}) (x)

And since μ_{zint} (min { $\mu_{\tilde{G}}$ (x), $\mu_{\tilde{B}}$ (x)}) ≤ min { $\mu_{\tilde{G}}$ (x), $\mu_{\tilde{B}}$ (x)}

Then we get Min { { $\mu_{\tilde{G}}$ (x), $\mu_{\tilde{B}}$ $(x$)}= μ_{zint} (min {$\mu_{\tilde{G}}$ (x), $\mu_{\tilde{B}}$ (x)}) (x)

Thus $\tilde{G} \cap \tilde{B}$ is a fuzzy Z-open set in Ã.

2.15 Proposition:

Let (Ã,$\tilde{T}$) be a fuzzy topological space , if $\tilde{C}$ be a fuzzy Z-open set in $\tilde{A}$ and $\tilde{B}$, is a maximal fuzzy open set in $\tilde{A}$, then $\tilde{C} \cap \tilde{B}$ is a fuzzy Z-open set in $\tilde{A}$

Proof : obvious

2.16 Proposition:

Let ($\tilde{A}$, $\tilde{T}$) is a fuzzy topological space , if $\tilde{B}$ is a fuzzy set in $\tilde{A}$, then $\tilde{G}$and is a fuzzy Z-open set in $\tilde{A}$, then $\tilde{G}$ $\tilde{\bar{q}}$ $\tilde{B}$ ⇔ $\tilde{G}$ $\tilde{\bar{q}}$ z-cl($\tilde{B}$) .

Proof

(⇒) Let $\tilde{G}$ $\tilde{\bar{q}}$ z-cl($\tilde{B}$) , since $\tilde{B}$ ≤ z-cl($\tilde{B}$)

Then $\tilde{G}$ $\tilde{\bar{q}}$ $\tilde{B}$ by remark (1.1.11)

(⟸) Suppose that $\tilde{G}$ $\tilde{\bar{q}}$ $\tilde{B}$

Then $\mu_{\tilde{G}}$(x) ≤ $\mu_{\tilde{B}^C}$ (x) by proposition (1.1.10)

And μ_{zint} ($\tilde{G}$) (x) ≤ $\mu_{zint\,(\tilde{B}^C)}$ (x) , since ($\tilde{G}$) is a fuzzy Z-open

Hence $\tilde{G}$ $\tilde{\bar{q}}$ z-cl($\tilde{B}$).

2.17 Definition [Hanafy, I. M., Benchalli,S.S.and Jenifer Karnel , A.M.Mubarkiali M.M.AL-Rshudi M.A.AL-Juhani**,** V.Seenivasan, K, Kamala**]**

A fuzzy set $\tilde{B}$ of a fuzzy topological space ($\tilde{A}$, $\tilde{T}$) is said to be :-

1) Fuzzy b- open (Fuzzy b- closed) set if

$\mu_{\tilde{B}} \leq \max \{ \mu_{int(cl(\tilde{B})}$ (x) , $\mu_{cl\,(int(\tilde{B})}$ (x) }.

$\mu_{\tilde{B}} \leq \min \{ \mu_{int(cl(\tilde{B})}$ (x) , $\mu_{cl\,(int(\tilde{B})}$ (x) }.

The family of all fuzzy B open set (fuzzy B closed) sets in $\tilde{A}$ will be denoted by FBO($\tilde{A}$) (FBO($\tilde{A}$)) .

2) Fuzzy β- open (Fuzzy β -closed) set if

$\mu_{\tilde{B}} \leq \{ \mu_{cl(int(cl(\tilde{B}))} \}$,

$\{\mu_{int(cl(int(\tilde{B})))}\} \leq \mu_{\tilde{B}}$

The family of all fuzzy β-open (fuzzyβ- closed) set in $\tilde{A}$ will be denoted byFβO ($\tilde{A}$) (FβC($\tilde{A}$)).

3) Fuzzy e-open (Fuzzy e-closed) set if

$\mu_{\tilde{B}} \leq \max \{ \mu_{clint_{\delta}(\tilde{B})}$(x) , $\mu_{int\,(cl_{\delta}(\tilde{B})}$(x) }.

$\mu_{\tilde{B}} \geq \min \{ \mu_{clint_{\delta}(\tilde{B})}$(x) , $\mu_{int\,(cl_{\delta}(\tilde{B})}$(x) }.

The family of all fuzzy e-open (fuzzy e-closed)set in $\tilde{A}$ will be denoted by FEO($\tilde{A}$) (FEC($\tilde{A}$)).

4) Fuzzyβ^*- open (Fuzzy β^*-closed) set if

$\mu_{\tilde{B}} \leq \max\{ \mu_{cl(int(cl(\tilde{B})}$(x) , $\mu_{int(cl_{\delta})}(\tilde{B})$ (x) }

$\mu_{\tilde{B}} \geq \min\{ \mu_{cl(int(cl(\tilde{B})}$(x), $\mu_{int(cl_{\delta})}(\tilde{B})$ (x) } }

The family of all Fuzzy β^*-open (Fuzzy β^*-closed) set in $\tilde{A}$ will be denoted by (Fβ^*O ($\tilde{A}$)) (F$\beta^* C$($\tilde{A}$)).

2.18 Proposition:

Let ($\tilde{A}, \tilde{T}$) be a fuzzy topological space then:

1. Every fuzzy Z-open set (resp. fuzzy Z-closed set) is fuzzy b-open set (resp. fuzzy b-closed set).
2. Every fuzzy Z-open set (resp. fuzzy Z-closed set) is fuzzy β- open set (resp. fuzzy β -closed set).
3. Every fuzzy Z-open set (resp. fuzzy Z-closed set) is fuzzy e - open set (resp. fuzzy e -closed set).
4. Every fuzzy Z-open set (resp. fuzzy Z-closed set) is fuzzy β^* open set (resp. fuzzy β^*-closed set)

proof :Obvious .

2.19 Remark:

The converse of proposition (2.13) is not true in general as following examples show:

2.20 Example

Let X={a ,b } and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.9),(b,0.9) }

$\tilde{B}$ ={ (a,0.3),(b,0.3) }

$\tilde{C}$ ={ (a,0.2),(b,0.2) }

$\widetilde{D}$ ={ (a,0.5),(b,0.5) }

$\tilde{E}$ ={ (a,0.4),(b,0.4) }

$\tilde{F}$ ={ (a,0.6),(b,0.5) }

Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$,$\widetilde{C}$,$\widetilde{D}$,$\tilde{E}$ } be a fuzzy topology on $\widetilde{A}$

Then the fuzzy set $\tilde{F}$ is a fuzzy b-open set but not fuzzy Z-open set

2.21Example:

Let X={a,b} and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.8),(b,0.8) }

$\tilde{B}$ ={ (a,0.4),(b,0.4) }

$\tilde{C}$ ={ (a,0.2),(b,0.2) }

$\widetilde{D}$ ={ (a,0.5),(b,0.4) }

$\tilde{E}$ ={ (a,0.6),(b,0.4) }

$\tilde{F}$ ={ (a,0.5),(b,0.5) }

Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$,$\widetilde{C}$,$\widetilde{D}$,$\tilde{E}$ } be a fuzzy topology on $\widetilde{A}$

Then the fuzzy set $\tilde{F}$ is a fuzzy β-open and fuzzy β^*-open but not fuzzy Z-open set.

2.22Example:

Let X={a,b} and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.6),(b,0.6) }

$\tilde{B}$ ={ (a,0.1),(b,0.3) }

$\tilde{C}$ ={ (a,0.4),(b,0.4) }

$\widetilde{D}$ ={ (a,0.4),(b,0.3) }

$\tilde{E}$ ={ (a,0.2),(b,0.2) }

$\tilde{F}$ ={ (a,0.5),(b,0.2) }

$\tilde{G}$={(a,0.2),(b,0.3) }

$\widetilde{H}$={(a,0.1),(b,0.2) }, Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\tilde{E}$,$\tilde{G}$, $\widetilde{H}$ } be a fuzzy topology on $\widetilde{A}$

Then the fuzzy set $\tilde{F}$ is fuzzy e-open set but not fuzzy Z-open set

2.23 Remark

This diagram explain the relation between fuzzy (b-open, β-open, β^*- open ,e-open)sets with fuzzy Z-open set.

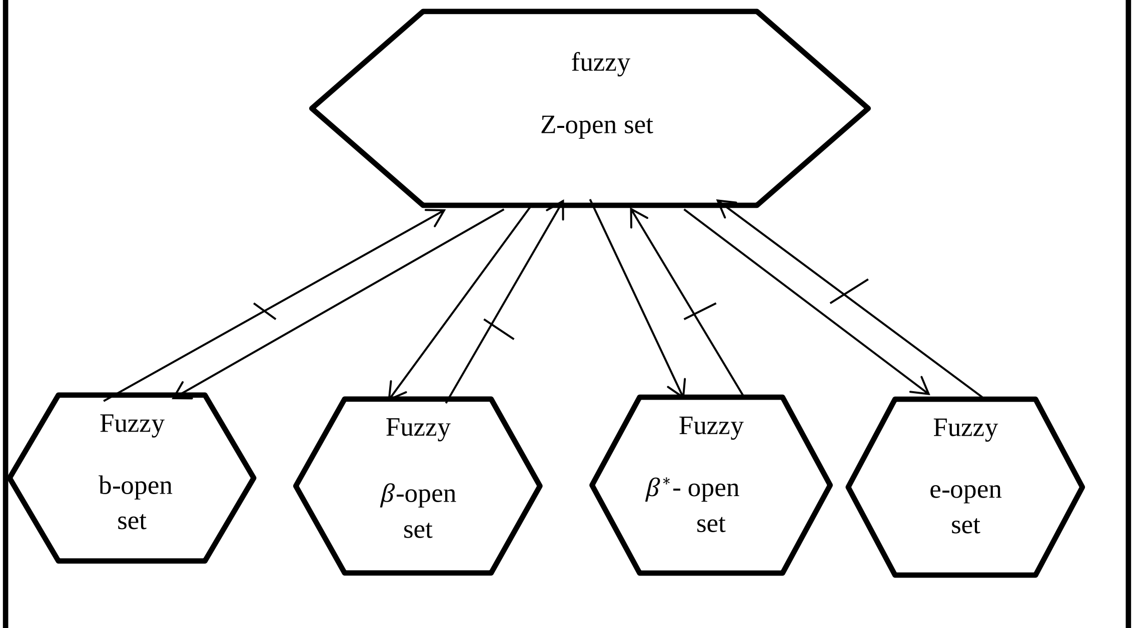

Diagram (1)

(2.24) Proposition:

Let ($\widetilde{A}$,$\widetilde{T}$) be a fuzzy topological space then:

1. Every fuzzy b-open set (resp. fuzzy b-closed set) is fuzzy β-open set (resp. fuzzy β –closed set) .
2. Every fuzzy β-open set (resp. fuzzy β-closed set) is fuzzy β^*-open set (resp. fuzzy β^*-open set).
3. Every fuzzy β^*-open set (resp. fuzzy β^*-closed set) is fuzzy e-open set (resp. fuzzy e-closed set).
4. Every fuzzy b-open set (resp. fuzzy b-closed set) is fuzzy e -open set (resp. fuzzy e–closed set) .

Proof: Obvious

(2.25) Remark:

The converse of proposition (1.4.2) is not true in general as following examples show:

2.26 Example

Let X={a,b} and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$,$\widetilde{G}$, $\widetilde{H}$,$\widetilde{w}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.7),(b,0.7) } , $\tilde{B}$ ={ (a,0.0),(b,0.4) }

$\tilde{C}$ ={ (a,0.4),(b,0.0) }, $\widetilde{D}$ ={ (a,0.4),(b,0.4) }

$\tilde{E}$ ={ (a,0.2),(b,0.2) }, $\tilde{F}$ ={ (a,0.0),(b,0.2) }

$\tilde{G}$={ (a,0.2),(b,0.4) } , $\widetilde{H}$={ (a,0.2),(b,0.0) }

$\widetilde{W}$ ={ (a,0.4),(b,0.2) } , $\widetilde{K}$ = { (a,0.0),(b,0.5) }

Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\widetilde{H}$, $\widetilde{W}$} be a fuzzy topology on $\tilde{A}$

Then the fuzzy set $\widetilde{K}$ is fuzzy β^*-open (resp. fuzzy b-open) but not fuzzy e-open set.

2.27Example:

Let X={a,b} and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.5),(b,0.5) }, $\tilde{B}$ ={ (a,0.2),(b,0.1) }

$\tilde{C}$ ={ (a,0.3),(b,0.2) }, $\widetilde{D}$ ={ (a,0.2),(b,0.2) }

$\tilde{E}$ ={ (a,0.3),(b,0.1) } , $\tilde{F}$ ={ (a,0.1),(b,0.3) }, Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\tilde{E}$ } be a fuzzy topology on $\widetilde{A}$

Then the fuzzy β- open set but not fuzzy b-open set.

2.28 Example:

Let X={a,b} and $\widetilde{B}$, $\tilde{C}$,$\widetilde{D}$,$\tilde{E}$,$\widetilde{F}$, $\widetilde{G}$, $\widetilde{H}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.6),(b,0.6) }, $\tilde{B}$ ={ (a,0.6),(b,0.1) }

$\tilde{C}$ ={ (a,0.0),(b,0.2) }, $\widetilde{D}$ ={ (a,0.5),(b,0.2) }

$\tilde{E}$ ={ (a,0.0),(b,0.1) } , $\tilde{F}$ ={ (a,0.6),(b,0.2) }

$\tilde{G}$={ (a,0.5),(b,0.1) } , $\widetilde{H}$={ (a,0.0),(b,0.4) }

Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$ } be a fuzzy topology on $\tilde{A}$

Then the fuzzy set $\widetilde{H}$ is fuzzy β^{*}-open set but not fuzzy β-open set.

2.29 Example

Let X={a,b} and $\widetilde{B}, \tilde{C}, \widetilde{D}, \tilde{E}, \widetilde{F}$ are fuzzy subset in $\tilde{A}$ where:

$\tilde{A}$ ={ (a,0.7),(b,0.7) }, $\tilde{B}$ ={ (a,0.5),(b,0.5) }

$\tilde{C}$ ={ (a,0.4),(b,0.4) } , $\widetilde{D}$ ={ (a,0.2),(b,0.2) }

$\tilde{E}$ ={ (a,0.3),(b,0.3) }

Let $\widetilde{T}$ ={$\widetilde{\emptyset}$,$\tilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$} be a fuzzy topology on $\widetilde{A}$ Then the $\tilde{E}$ fuzzy e-open set but not fuzzy b-open set.

2.30Remark

This diagram explain the relation between some type of fuzzy (b-open, β-open, β^*- open ,e-open)sets with fuzzy Z-open set

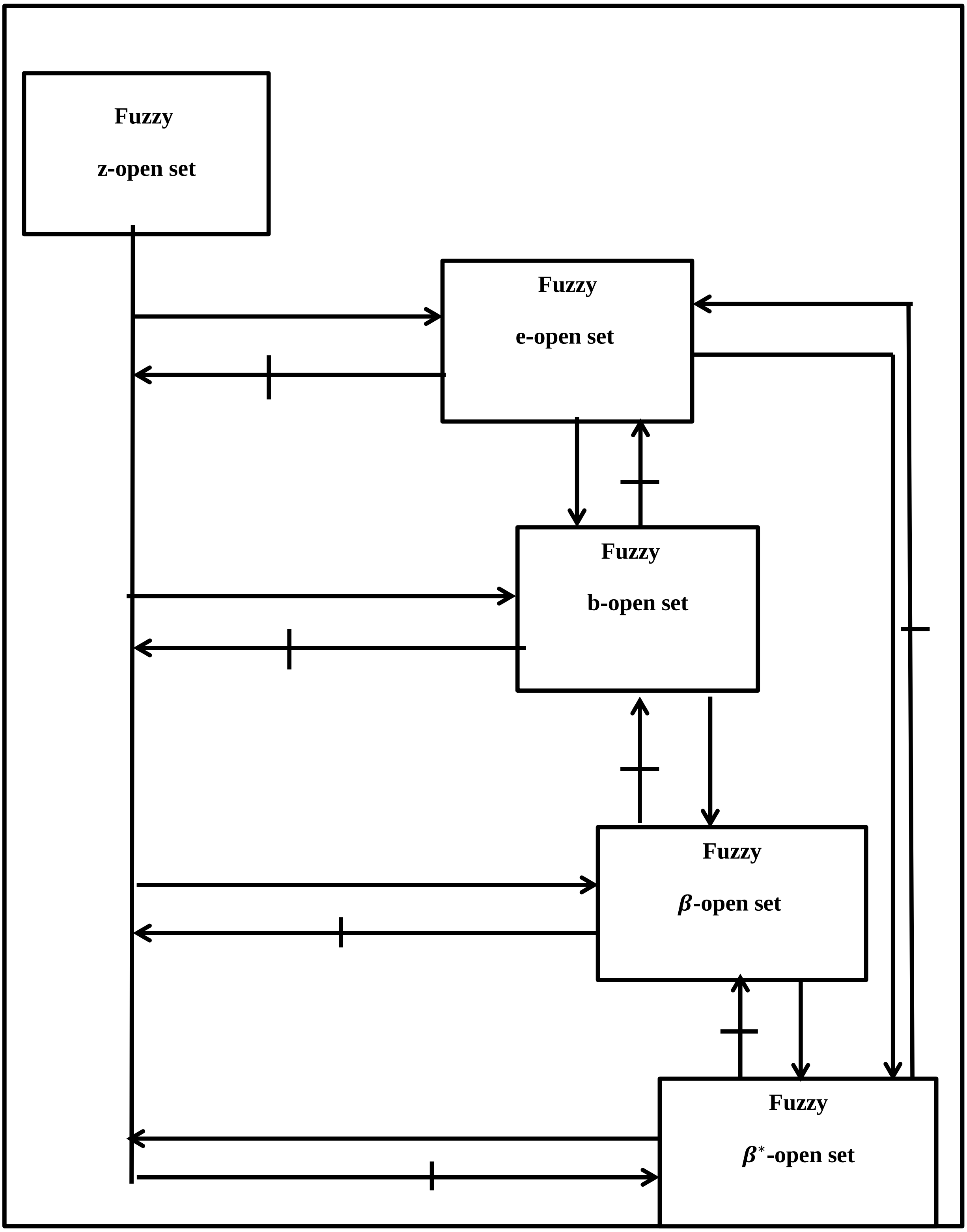

Diagram (2)

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