On Some Types of Fuzzy ɵ - Continuous Functions in Fuzzy Topological Space on Fuzzy Set

Assist .Prof. Dr. Munir Abdul Khalik AL-Khafaji, Heba Mahdi Mousa

AL-Mustinsiryah University \ Collage of Education / Department of Mathematics

Abstract: The aim of this paper is to introduce fuzzy ɵ -open set and to find different characterization of fuzzy ɵ -continuous function and to show the relationships between them ,where we confine our study to some of their types and giving some properties and theorems related to it.

I. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper in 1965.The fuzzy Topological space was introduced by chang in 1968.Chakrabarty and Ahsanullah introduced the notion of fuzzy topological space on fuzzy set.M.E.El.Shafei and A.Zakeri defined fuzzy ɵ -open sets. the concept of fuzzy ɵ − continuous has been introduced by Yalvas ,Mukherjee and Sinha . fuzzy almost continuous function which had been defined by Azad .In this paper we introduce fuzzy ɵ -open set and study some kind of fuzzy ɵ -continuous function and the relationships between them.

1. fuzzy topological space on fuzzy set

1.1 Definition [Zadeh, L.A**] :**Let X be a non empty set, a fuzzy set Ã in X is characterized by a function

$\mu_{\tilde{A}} : X \to I$, where $I = [0,1]$ which is written as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, 0 \le \mu_{\tilde{A}}(x) \le 1\}$, the collection of all fuzzy sets in X will be denoted by I^X,that is

$I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where $\mu_{\tilde{A}}$ is called the membership function

1.2 Proposition [Wong, C. K**] :**

Let Ã and B̃ be two fuzzy sets in X with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively,

then for all $x \in X$: -

1. $\mu_{\tilde{\Phi}}(x) = \mathbf{0}$
2. $\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$.
3. $\tilde{A} = \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
4. $\tilde{C} = \tilde{A} \cap \tilde{B}$ if and only if $\mu_{\tilde{C}}(x) = \min\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
5. $\tilde{D} = \tilde{A} \cup \tilde{B}$ if and only if $\mu_{\tilde{D}}(x) = \max\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.

1.3 Definition [Kandil1 A, S. Saleh2 and M.M Yakout3**] :**A fuzzy point x_r is a fuzzy set such that :

$\mu_{x_r}(y) = r > 0$ if $x = y$, $\forall y \in X$ and

$\mu_{x_r}(y) = 0$ if $x \neq y$, $\forall y \in X$

The family of all fuzzy points of Ã will be denoted by FP(Ã)

1.4 Remark **[**Chakraborty M. K. And T. M. G. Ahsanullah**] :**
Let $\tilde{A} \in I^X$, then $P(\tilde{A}) = \{ \tilde{B} : \tilde{B} \in I^X, \tilde{B} \subseteq \tilde{A} \}$

1.5 Definition **[**Mahmoud F. S, M. A. Fath Alla, and S. M. Abd Ellah,**] :**If $\tilde{B} \in (\tilde{A},\tilde{T})$,the complement of $\tilde{B}$ referred to $\tilde{A}$ denoted by $\tilde{B}^c$ is defined by

$$\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x) \ \forall \ x \in X$$

1.6 Definition [Chang, C. L**] :**A collection $\tilde{T}$ of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfied the following conditions

1. $\tilde{A} \ , \tilde{\phi} \ \in \tilde{T}$
2. If $\tilde{B} , \tilde{C} \in \tilde{T}$, then $\tilde{B} \cap \tilde{C} \ \in \tilde{T}$
3. If $\tilde{B}_\alpha \in \tilde{T}$, then $\bigcup_\alpha \tilde{B}_\alpha \ \in \tilde{T} , \ \alpha \in \Lambda$

$(\tilde{A},\tilde{T})$ is said to be Fuzzy topological space and every member of $\tilde{T}$ is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set

1.7 Definition **[**Balasubramanion G.**] :**A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A} ,\tilde{T})$ is said to be a fuzzy neighborhood of a fuzzy point x_r in $\tilde{A}$ if there is a fuzzy open set $\tilde{G}$ in $\tilde{A}$ such that
, $x_r \subseteq \tilde{G} \subseteq \tilde{B}$

1.8 Definitions **[**Bai Shi – Zhong, Wang Wan – Liang**,** Balasubramanion G.**] :**
Let $\tilde{B}$, $\tilde{C}$ be a fuzzy set in a fuzzy topological space $(\tilde{A} , \tilde{T})$ then:

- A fuzzy point x_r is said to be quasi coincident with the fuzzy set $\tilde{B}$ if there exist $x \in X$ such that $\mu_{x_r}(x) + \mu_{\tilde{B}}(x) > \mu_{\tilde{A}}(x)$ and denote by $x_r \, q \, \tilde{B}$, if $\mu_{x_r}(x) + \mu_{\tilde{B}}(x) \leq \mu_{\tilde{A}}(x) \ \forall \ x \in X$ then x_r is not quasi coincident with a fuzzy set $\tilde{B}$ and is denoted by $x_r \, \tilde{q} \, \tilde{B}$.
- A fuzzy set $\tilde{B}$ is said to be quasi coincident (overlap) with a fuzzy set $\tilde{C}$ if there exist $x \in X$ such that $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) > \mu_{\tilde{A}}(x)$ and denoted by $\tilde{B} \, q \, \tilde{C}$, if $\mu_{\tilde{B}}(x) + \mu_{\tilde{C}}(x) \leq \mu_{\tilde{A}}(x) \ \forall \ x \in X$ then $\tilde{B}$ is not quasi coincident with a fuzzy set $\tilde{C}$ and is denoted by $\tilde{B} \, \tilde{q} \, \tilde{C}$.

2. fuzzy θ-open set

2.1Definition **:**A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A} ,\tilde{T})$ is said to be **Fuzzy θ-open set** if for each x_r in $\tilde{B}$ there exist an fuzzy open set $\tilde{C}$ containing x_r such that $Cl(Int(\tilde{C})) \subseteq \tilde{B}$, $\tilde{B}$ is called **[Fuzzy θ-closed]** set if its complement is Fuzzy θ-open set.The family of all Fuzzy θ-open (Fuzzy θ-closed) sets in $\tilde{A}$ will be denoted by $F\theta O(\tilde{A})$ ($F\theta C(\tilde{A})$).

2.2 Example :

Let $X = \{ a , b , c \}$ and $\tilde{B}$, $\tilde{C}$ be fuzzy subsets of $\tilde{A}$ where:

$\tilde{A} = \{(a ,0.7) ,(b ,0.7) , (c ,0.6)\}$,

$\tilde{B} = \{(a ,0.3) , (b ,0.2) , (c ,0.2)\}$,

$\tilde{C} = \{(a ,0.4) , (b , 0.5) , (c ,0.4)\}$,

Let $\tilde{T} = \{ \tilde{\emptyset} , \tilde{A} , \tilde{B} , \tilde{C} \}$, be a fuzzy topologies on $\tilde{A}$,

Then the fuzzy set $\tilde{B}$ in fuzzy topological space $(\tilde{A}, \tilde{T})$ is fuzzy θ-open set.

2.3 Definition [Seok Jonng and Sang Min Yun**] :** A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A} ,\tilde{T})$ is called fuzzy θ-quasi neighborhood of a fuzzy point x_r in $\tilde{A}$ if there is a fuzzy θ-open set $\tilde{G}$ in $\tilde{A}$ such that $x_r \, q \, \tilde{G}$ and $\tilde{G} \subseteq \tilde{B}$

2.4 Proposition :

Let $\tilde{B}$ and $\tilde{C}$ be fuzzy sets in a fuzzy topological space $(\tilde{A}, \tilde{T})$, then ;

1. $\mu_{\theta cl(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{\theta cl(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$.
2. If $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}}(x)$ then $\mu_{\theta cl(\tilde{B})}(x) \leq \mu_{\theta cl(C)}(x)$.
3. $\mu_{\tilde{B}}(x) \leq \mu_{\theta cl(\tilde{B})}(x)$.
4. $\mu_{\theta cl(\theta cl(\tilde{B}))}(x) = \mu_{\theta cl(\tilde{B})}(x)$.
5. $\mu_{\theta cl(\min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x) \leq \min\{ \mu_{\theta cl(\tilde{B})}(x) , \mu_{\theta cl(C)}(x) \}$.
6. $\mu_{\theta cl(\max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x) = \max\{ \mu_{\theta cl(\tilde{B})}(x) , \mu_{\theta cl(C)}(x) \}$.

Proof :obvious

2.5 Proposition: Let $(\tilde{A}, \tilde{T})$ be a fuzzy topological space and $\tilde{B} \in P(\tilde{A})$, Then
$\mu_{\theta in(\tilde{\emptyset})}(x) = \mu_{(\theta cl(\tilde{B}^c))^c)}(x)$

Proof :

Since $\mu_{\theta int(\tilde{B})}(x) \leq \mu_{\tilde{B}}(x)$

And θint$(\tilde{B})$ is a fuzzy θ-open set

Then $\mu_{\tilde{B}^c}(x) \leq \mu_{(\theta int(\tilde{B}))^c}(x)$ and $\mu_{\theta cl(\tilde{B}^c)}(x) \leq \mu_{(\theta int(\tilde{B}))^c}(x)$, hence

$\mu_{\theta int(\tilde{B})}(x) \leq \mu_{(\theta cl(\tilde{B}^c))^c}(x)$ ……….(*)

Since $\mu_{\tilde{B}^c}(x) \leq \mu_{\theta cl(\tilde{B}^c)}(x)$ and $\theta cl(\tilde{B})$ is a fuzzy θ-closed set then $\mu_{(\theta cl(\tilde{B}^c))^c}(x) \leq \mu_{\tilde{B}}(x)$

Hence $\mu_{(\theta cl(\tilde{B}^c))^c}(x) \leq \mu_{\theta int(\tilde{B})}(x)$ …………….(**)

From (*) and (**) we get $\mu_{\theta int(\tilde{B})}(x) = \mu_{(\theta cl(\tilde{B}^c))^c}(x)$ ■

3.some types of fuzzy θ-continuous function

3.1Definition [Zadeh, L.A.,**]:**Let f be a function from $(\tilde{A}, \tilde{T})$ to $(\tilde{B}, \tilde{T})$. Let $\tilde{D}$ be a fuzzy subset in $\tilde{B}$ with membership function $\mu_{\tilde{D}}(\tilde{B})$ Then, the inverse of $\tilde{D}$ written as $f^{-1}(\tilde{D})$, is a fuzzy subset of $\tilde{A}$ whose membership function defined by: $\mu_{f^{-1}(\tilde{D})}(x) = \mu_{\tilde{D}}(f(x))$, for all x in $\tilde{A}$

Conversely, let $\tilde{C}$ be a fuzzy subset in $\tilde{A}$ with membership function $\mu_{\tilde{C}}(\tilde{A})$. The image of $\tilde{C}$ written as $f(\tilde{C})$, is a fuzzy subset in $\tilde{B}$ whose membership function is given by:

For all $y \in \tilde{B}$., where $f^{-1}(y) = \{x : f(x) = y\}$

3.2 Theorem [Qutaiba Ead Hassan**]**

Let f be a function from $\tilde{A}$ to $\tilde{B}$, and J_X be any index set. The following statements are true.

1. If $\tilde{C} \subset \tilde{A}$, then $f(\tilde{C})^c \subset f(\tilde{C}^c)$.
2. If $\tilde{D} \subset \tilde{B}$, then $f(\tilde{D})^c \subset f(\tilde{D}^c)$.
3. If $\tilde{C}_1, \tilde{C}_2 \subset \tilde{A}$ and $\tilde{C}_1 \subset \tilde{C}_2$,then f $(\tilde{C}_1) \subset f(\tilde{C}_2)$
4. If $\tilde{D}_1, \tilde{D}_2 \subset \tilde{B}$ and $\tilde{D}_1 \subset \tilde{D}_2$,then $f^{-1}(\tilde{D}_1) \subset f^{-1}(\tilde{D}_2)$
5. If $\tilde{C} \subset \tilde{A}$, then $\tilde{C} \subset f^{-1}(f(\tilde{C}))$.

6. If $\tilde{D} \subset \tilde{B}$, then $f(f^{-1}(\tilde{D})) \subset \tilde{D}$

7. If $\tilde{C}_i \subset \tilde{A}$, for every $i \in I$,then f $U_{i \in I} \tilde{A}_i = U_{i \in I} \tilde{A}_i$

8. If $\tilde{D}_i \subset \tilde{B}$ for every $i \in I$, then f^{-1} $I_{i \in I} \tilde{B}_i = I_{i \in I} f^{-1}(\tilde{B}_i)$

9.If $\tilde{D}_i \subset \tilde{B}$, for every $i \in I$, thenf^{-1} $U_{i \in I} \tilde{B}_i = U_{i \in I} f^{-1}(\tilde{B}_i)$

10.If f is onto and $\tilde{D} \subset \tilde{B}$,then $f(f^{-1}(\tilde{D})) = \tilde{D}$

11.If $\tilde{C}, \tilde{D} \subset \tilde{A}$, then $f(\tilde{C} \cap \tilde{D}) \subseteq f(\tilde{C}) \cap f(\tilde{D})$.

12.Let f be a function from $\tilde{A}$ to $\tilde{B}$ and g a function from $\tilde{B}$ to Z. If $\tilde{D} \subset Z$, then $(gof)^{-1}(\tilde{D}) = f^{-1}(g^{-1}(\tilde{D}))$;

if $\tilde{C} \subset \tilde{A}$, then $(gof)(\tilde{C}) = g(f(\tilde{C}))$

3.3 Definition [Chakraborty M. K. And T. M. G. Ahsanullah**]:**A function $f : (\tilde{A}, \tilde{T}) \longrightarrow (\tilde{B}, \tilde{U})$ is **fuzzy continuous** (F-continuous) if and only if the inverse image of each fuzzy$\tilde{U}$-open set is fuzzy

$\tilde{T}$ -open set.

3.4 Definition [Park ,J.H.,Lee,B.Y.and Choi,J.R**]:**A function $f : (\tilde{A}, \tilde{T}) \longrightarrow (\tilde{B}, \tilde{T}')$ is said to be

1.fuzzy θ-continuous (f.θ.c, for short) if for each fuzzy point x_r in $(\tilde{A}, \tilde{T})$ and each fuzzy open q-nbd. of $\tilde{V}$ $f(x_r)$, there exists fuzzy open q-nbd. $\tilde{U}$ of x_r such that $f(cl(\tilde{U})) \subseteq (cl(\tilde{V}))$.

2.fuzzy strong θ-continuous (f.s.θ.c, for short) if for each fuzzy point x_r in $(\tilde{A}, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(x_r)$, there exists fuzzy open q-nbd. $\tilde{U}$of x_r such that $f(cl(\tilde{U})) \subseteq (\tilde{V})$.

3. fuzzy weak θ-continuous (f.s.θ.c, for short) if for each fuzzy point x_r in $(\tilde{A}, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(x_r)$, there exists fuzzy open q-nbd. $\tilde{U}$of x_r such that $f(int(cl(\tilde{U}))) \subseteq cl(\tilde{V})$

***4.* fuzzy super θ-continuous** (f.s.θ.c, for short) if for each fuzzy point x_r in $(\tilde{A}, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(x_r)$, there exists fuzzy open q-nbd. $\tilde{U}$of x_r such that $f(int(cl(\tilde{U}))) \subseteq (\tilde{V})$

***5.* fuzzy almost-continuous** (f.s.θ.c, for short) if for each fuzzy point x_r in $(\tilde{A}, \tilde{T})$ and each fuzzy open q-nbd. $\tilde{V}$ of $f(x_r)$, there exists fuzzy open q-nbd. $\tilde{U}$ of x_r such that $f(\tilde{U}) \subseteq int(cl(\tilde{V}))$.

3.5 Proposition: Every fuzzy strong θ-continuous function is fuzzy super θ-continuous function.

Proof:

Let a function $f : (\tilde{A}, \tilde{T}) \longrightarrow (\tilde{B}, \tilde{T}')$ be fuzzy strong θ-continuous function. By definition of fuzzy strong θ-continuous function, $\mu_{f(cl(\tilde{U}))}(x) \leq \mu_{(\tilde{V})}(x)$.

$\mu_{f(int(cl(\tilde{U})))}(x) \leq \mu_{int(\tilde{V})}(x) = \mu_{(\tilde{V})}(x)$.

Hence $\mu_{f(int(cl(\tilde{U})))}(x) \leq \mu_{(\tilde{V})}(x)$ Then by definition of fuzzy super θ-continuous function f is fuzzy super θ-continuous.

3.6 Proposition: Every fuzzy super θ-continuous function is fuzzy continuous function.

Proof :obvious

3.7 Remark :The converse of proposition(***3.6***) is not true in general as the following example shows:-

3.8 Example:

Let x be any non-empty set and a,b ∈x

Let $\tilde{T}=\{\tilde{A}, \tilde{\Phi}, \tilde{B}, \tilde{C}, \widetilde{D}\}$, $\acute{T} = \{\tilde{A}, \tilde{\Phi}, \tilde{C}\}$

$\tilde{A} = \{(a, 0.9), (b, 0.9)\}$

$\tilde{B} = \{(a, 0.4), (b, 0.4)\}$ $\tilde{B}^c = \{(a, 0.5), (b, 0.5)\}$

$\tilde{C} = \{(a, 0.3), (b, o.3)\}$ $\tilde{C}^c = \{(a, 0.6), (b, o.6)\}$

$\widetilde{D} = \{(a, 0.2), (b, 0.2)\}$ $\widetilde{D}^c = \{(a, 0.7), (b, 0.7)\}$

Let f : $(\tilde{A}, \tilde{T}) \longrightarrow (\tilde{A}, \acute{T})$ be the identity function

Let $x_r = \{(a, 0)(b, 0.1)\}$ then $\widetilde{D}$ is fuzzy open q-nbhd of x_r in $(\tilde{A}, \tilde{T})$

Such that $\mu_{cl(\widetilde{D})}(x) = \mu_{\tilde{B}^c}(x) \rightarrow \mu_{int(cl(\widetilde{D})}(x) = \mu_{int(\tilde{B}^c)}(x) = \mu_{(\tilde{B})}(x)$.

And $\mu_{f(int(cl(\widetilde{D}))}(x) = \mu_{(\tilde{B})}(x) \nleq \mu_{(\tilde{C})}(x)$.

$\tilde{C}$ is fuzzy q-nbhd of x_r in$(\tilde{A}, \acute{T})$ hence f is continuous function but f is not supper continuous function.

3.9 Proposition: Every fuzzy continuous function is fuzzy almost-continuous function.

Proof :obvious

3.10 Remark:The converse of proposition(3.9) is not true in general as the following example shows:-

3.11 Example:

Let x be any non-empty set and a,b ∈x any fixed element

let $\tilde{T}=\{\tilde{A}, \tilde{\Phi}, \tilde{B}, \tilde{C}, \}$, $\acute{T} = \{\tilde{A}, \tilde{\Phi}, \widetilde{D}, \tilde{C}\}$

$\tilde{A} = \{(a, 0.9), (b, 0.9)\}$

$\tilde{B} = \{(a, 0.3), (b, 0.4)\}$ $\tilde{B}^c = \{(a, 0.6), (b, 0.5)\}$

$\tilde{C} = \{(a, 0.6), (b, o.5)\}$ $\tilde{C}^c = \{(a, 0.3), (b, o.4)\}$

$\widetilde{D} = \{(a, 0.2), (b, 0.1)\}$ $\widetilde{D}^c = \{(a, 0.7), (b, 0.8)\}$

Let f : $(\tilde{A}, \tilde{T}) \longrightarrow (\tilde{A}, \acute{T})$ be the identity function

$\mu_{(int(cl(\tilde{B}))}(x) = \mu_{(\tilde{B})}(x)$

$\mu_{(int(cl(\tilde{C}))}(x) = \mu_{(\tilde{C})}(x)$

Therefore $\tilde{B}, \tilde{C}$ are fuzzy regular open sets of $(\tilde{A}, \acute{T})$while $\widetilde{D}$ is not, and we have $\mu_{f(\tilde{\Phi})} = \mu_{(\tilde{\Phi})}$, $\mu_{f(\tilde{B})} = \mu_{(\tilde{B})}$, $\mu_{f(\tilde{C})}(x) = \mu_{(\tilde{C})}(x)$,Such that $\mu_{f(\tilde{B})}(x) \leq \mu_{(int(cl(\tilde{B}))}(x) = \mu_{(\tilde{B})}$ similary with $\tilde{C}$

It is obvious that f is a fuzzy almost-continuous function.

Since $\mu_{f^{-1}(\widetilde{D})}(x) = \mu_{(\widetilde{D})}(x)$ but $\widetilde{D} \notin (\tilde{A}, \tilde{T})$ so f is not fuzzy continuous function.

3.12 Proposition: Every fuzzy almost-continuous function is fuzzy θ-continuous function.

Proof :obvious

3.13 Remark: The converse of proposition(3.12) is not true in general as the following example shows:-

3.14 Example:

let $\tilde{T}=\{\tilde{A}, \tilde{\Phi}, \tilde{B}\}$, $\acute{\tilde{T}}=\{\tilde{A}, \tilde{\Phi}, \tilde{D}, \tilde{C}\}$

$\tilde{A}=\{(a, 0.9),(b, o.9)\}$

$\tilde{B}=\{(\mathrm{a}, 0.5),(\mathrm{b}, 0.2)\}$ $\quad \tilde{B}^c=\{(\mathrm{a}, 0.4),(\mathrm{b}, 0.7)\}$

$\tilde{C}=\{(a, 0.4),(b, o.4)\}$ $\quad \tilde{C}^c=\{(a, 0.5),(b, o.5)\}$

$\tilde{D}=\{(\mathrm{a}, 0.4),(\mathrm{b}, 0.7)\}$ $\quad \tilde{D}^c=\{(\mathrm{a}, 0.5),(\mathrm{b}, 0.2)\}$

Let f : $(\tilde{A}, \tilde{T}) \longrightarrow (\tilde{A}, \acute{\tilde{T}})$ be the identity function

$\mu_{(\mathrm{cl}(\tilde{\mathrm{B}}))}(\mathrm{x})=\mu_{(\tilde{D}^c)}(\mathrm{x})$ $\quad \mu_{(\mathrm{cl}(\tilde{\mathrm{c}}))}(\mathrm{x})=\mu_{(\tilde{C}^c)}(\mathrm{x})$

Let x_r be a fuzzy point in $(\tilde{A}, \tilde{T})$ and $\tilde{V}$ be any fuzzy open q-nbd of f(x_r)in $(\tilde{A}, \acute{\tilde{T}})$ if $\mu_{(\tilde{V})}(\mathrm{x})=\mu_{(\tilde{C})}(\mathrm{x})$ then we choose $\mu_{(\tilde{U})}(\mathrm{x})=\mu_{(\tilde{B})}(\mathrm{x})$ and then $\tilde{U}$ Is fuzzy open q-nbd of(x_r) such that

$\mu_{\mathrm{f}(\mathrm{cl}(\tilde{\mathrm{U}}))}(\mathrm{x})=\mu_{\mathrm{f}(\mathrm{cl}(\tilde{\mathrm{B}}))}(\mathrm{x})=\mu_{\mathrm{f}(\tilde{D}^c)}(\mathrm{x})=\mu_{(\tilde{D}^c)}(\mathrm{x})$ and $\mu_{(\mathrm{cl}(\tilde{\mathrm{V}}))}(\mathrm{x})=\mu_{(\mathrm{cl}(\tilde{\mathrm{C}}))}(\mathrm{x})=\mu_{(\tilde{C}^c)}(\mathrm{x})$ Since

$\mu_{(\tilde{D}^c)}(\mathrm{x}) \leq \mu_{(\tilde{C}^c)}(\mathrm{x})$ hence $\mu_{\mathrm{f}(\mathrm{cl}(\tilde{\mathrm{U}}))}(\mathrm{x}) \leq \mu_{(\mathrm{cl}(\tilde{\mathrm{V}}))}(\mathrm{x})$

So f is fuzzy θ-continuous function.

Now $\tilde{C}$ is fuzzy open q-nbd of f(x_r) in $(\tilde{A}, \acute{\tilde{T}})$ and $\tilde{B}, \tilde{A}$ are fuzzy open q-nbd of f(x_r)in$(\tilde{A}, \tilde{T})$ .since $\mu_{\mathrm{f}(\tilde{\mathrm{B}})}(\mathrm{x})=\mu_{(\tilde{\mathrm{B}})}(\mathrm{x})$ $\quad \mu_{(\mathrm{cl}(\tilde{\mathrm{C}}))}(\mathrm{x})=\mu_{(\tilde{C}^c)}(\mathrm{x})$ And $\mu_{(\mathrm{int}(cl(\tilde{C}))}(\mathrm{x})=\mu_{(\mathrm{int}(\tilde{C}))}(\mathrm{x})=\mu_{(\tilde{\mathrm{C}})}(\mathrm{x})$ there for $\mu_{\mathrm{f}(\tilde{\mathrm{B}})}(\mathrm{x}) \nleq \mu_{(\mathrm{int}(cl(\tilde{C}))}(\mathrm{x})$

Hence f is not fuzzy almost-continuous function.

3.15 Proposition: Every fuzzy θ-continuous function is fuzzy weak θ-continuous function.

Proof :obvious

3.16 Remark: The converse of proposition(3.15) is not true in general as the following example shows:-

3.17 Example:

Let x be any non-empty set and a,b ∈x any fixed element

let $\tilde{T}=\{\tilde{A}, \tilde{\Phi}, \tilde{B}\}$, $\acute{\tilde{T}}=\{\tilde{A}, \tilde{\Phi}, \tilde{C}\}$

$\tilde{A}=\{(a, 0.9),(b, 0.9)\}$

$\tilde{B}=\{(\mathrm{a}, 0.6),(\mathrm{b}, 0.7)\}$ $\quad \tilde{B}^c=\{(\mathrm{a}, 0.3),(\mathrm{b}, 0.2)\}$

$\tilde{C}=\{(a, 0.4),(b, o.2)\}$ $\quad \tilde{C}^c=\{(a, 0.5),(b, o.6)\}$

Let f : $(\tilde{A}, \tilde{T}) \longrightarrow (\tilde{A}, \acute{\tilde{T}})$ be the identity function

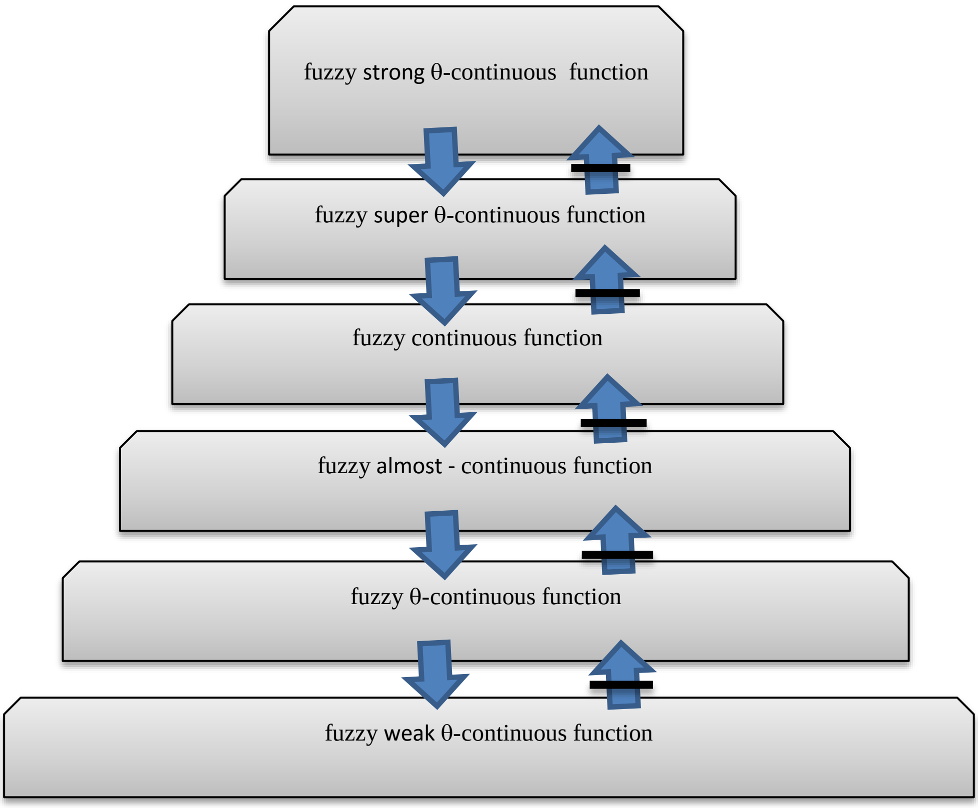
$\mu_{(\mathrm{cl}(\tilde{\mathrm{B}}))}(\mathrm{x})=\mu_{(\tilde{C}^c)}(\mathrm{x})$, $\mu_{(\mathrm{cl}(\tilde{\mathrm{C}}))}(\mathrm{x})=\mu_{(\tilde{B}^c)}(\mathrm{x})$

Let us consider x_r be a fuzzy point in$(\tilde{A}, \tilde{T})$ now $\tilde{C}$ is a is fuzzy open q-nbd of f(x_r) in $(\tilde{A}, \acute{\tilde{T}})$ since $\mu_{(\mathrm{cl}(\tilde{\mathrm{B}}))}(\mathrm{x})=\mu_{(\tilde{\mathrm{B}})}(\mathrm{x})$ $\quad \mu_{(\mathrm{cl}(\tilde{\mathrm{B}}))}=\mu_{(\tilde{C}^c)}$

$\mu_{int(cl(\tilde{B}))}(x) = \mu_{(\tilde{B}),}(x)$ and $\mu_{(cl(\tilde{C}))}(x) = \mu_{(\tilde{B}^c)}(x)$

$\mu_{f(int(cl(\tilde{B}))}(x) \leq \mu_{(\tilde{B})}(x)$ hence f is fuzzy weak θ-continuous function

But $\mu_{f(cl(\tilde{B}))}(x) \nleq \mu_{(cl(\tilde{C}))}(x)$ hence f is not fuzzy θ-continuous function



3.18 Theorem:

For a function $f : (\tilde{A}, \tilde{T}) \longrightarrow (\tilde{B}, \acute{\tilde{T}})$, each of the following statements equivalent:

(a) f is fuzzy θ-continuous function.

(b) For each fuzzy set $\tilde{C}$ in $\tilde{A}$, $f(\theta cl(\tilde{C})) \subseteq (\theta cl f(\tilde{C})$

(c) For each fuzzy set $\tilde{D}$ in $\tilde{B}$, $\theta cl f^{-1}(\tilde{D}) \subseteq f^{-1}\theta cl(\tilde{D})$

(d) For each θ-closed set $\tilde{D}$ in $\tilde{B}$, $f^{-1}(\tilde{D})$ is a fuzzy θ-closed in $\tilde{A}$.

(e) For each θ-open set $\tilde{D}$ in $\tilde{B}$, $f^{-1}(\tilde{D})$is a fuzzy θ-open in $\tilde{A}$.

(f) For each open set $\tilde{D}$ in $\tilde{B}$, $\theta clf^{-1}(\tilde{D}) \subseteq f^{-1}cl(\tilde{D})$

Proof:

(a) $\Rightarrow$ (b). Let $\mu_{(x_r)} \leq \mu_{\theta cl(\tilde{C})}$ be and let $\tilde{G}$ any open q-nbd. of $f(x_r)$. Then there exists an open q-nbd. $\tilde{V}$ of x_r such that $\mu_{fcl(\tilde{V})} \leq \mu_{cl(\tilde{G})}$ i.e., f is a fuzzy θ-continuous.

Now $\mu_{(x_r)} \leq \mu_{\theta cl(\tilde{A})}$, $cl(\tilde{V})$ q $\tilde{C} \Rightarrow f(cl(\tilde{V}))$ q $f(\tilde{C})$

$cl(\tilde{G})$ q $f(\tilde{C})$, $\mu_{f(x_r)} \leq \mu_{\theta clf(\tilde{C})}$, $\mu_{(x_r)} \leq \mu_{f^{-1}\theta cl(\tilde{C})}$, Thus, $\mu_{\theta cl(\tilde{C})} \leq \mu_{f^{-1}\theta cl(\tilde{C})}$

so that $\mu_{f(\theta cl(\tilde{C}))} \leq \mu_{\theta clf(\tilde{C})}$

(b) $\Rightarrow$ (c). By (b) $\mu_{f\theta(clf^{-1}(\tilde{D})} \leq \mu_{\theta clf(f^{-1}(\tilde{D})} \leq \mu_{\theta clf(\tilde{D})}$

Which implies that $\mu_{\theta cl(f^{-1}(\tilde{D})} \leq \mu_{f^{-1}\theta cl(\tilde{D})}$

(c) $\Rightarrow$ (d). We have $\mu_{\theta cl(\tilde{D})} = \mu_{(\tilde{D})}$

Now by (c) $\mu_{\theta cl(f^{-1}(\tilde{D})} \leq \mu_{f^{-1}\theta cl(\tilde{D})} = \mu_{f^{-1}(\tilde{D})}$

$\mu_{\theta(clf^{-1}(\tilde{D})} \leq \mu_{f^{-1}(\tilde{D})}$

For each fuzzy θ-closed set $\tilde{D}$ in $\tilde{B}$. $f^{-1}(\tilde{D})$ is a fuzzy θ-closed in $\tilde{A}$.

(e) $\Rightarrow$ (f). Since $\tilde{D}$ be a fuzzy open in $\tilde{B}$, $\mu_{cl(\tilde{D})} = \mu_{\theta cl(\tilde{D})}$ and we have from (c) $\mu_{\theta cl(f^{-1}(\tilde{D})} \leq \mu_{f^{-1}\theta cl(\tilde{D})}$.

We have $\mu_{\theta cl(f^{-1}(\tilde{D})} \leq \mu_{f^{-1}cl(\tilde{D})}$

(g) $\Rightarrow$ (a). Let x_r be a fuzzy point in $\tilde{A}$ and $\tilde{V}$ be a fuzzy open q-nbd. of $f(x_r)$. Then $\mu_{1-cl(\tilde{V})}$ (x) is a fuzzy open in $\tilde{B}$. By (f), we have $\mu_{\theta cl(f^{-1}(\tilde{A}-cl(\tilde{V})} \leq \mu_{f^{-1}cl(\tilde{A}-cl(\tilde{V}))} = \mu_{\tilde{A}-f^{-1}(int(cl(\tilde{V}))}$

Then there exists a fuzzy open q-nbd. $\tilde{G}$ of x_r such that

$cl(\tilde{G})\, \tilde{\bar{q}}\, (\tilde{A} - f^{-1}(cl(\tilde{V})))$ so that $\mu_{fcl(\tilde{G})} \leq \mu_{cl(\tilde{V})}$

Hence, f is a fuzzy θ-continuous function. ■

3.19Theorem:

For a function $f : (\tilde{A}, \tilde{T}) \longrightarrow (\tilde{B}, \tilde{T}')$, the following are equivalent:

(a) f is fuzzy strong θ-continuous.

(b) $f(\theta cl(\tilde{C})) \subseteq (clf(\tilde{C}))$ for every fuzzy set $\tilde{C}$ in $\tilde{A}$.

(c) $\theta clf^{-1}(\tilde{D}) \subseteq f^{-1}cl(\tilde{D})$ for every fuzzy set $\tilde{D}$ in $\tilde{B}$.

(d) The inverse image of every fuzzy closed set in $\tilde{B}$ is fuzzy θ-closed in $\tilde{A}$, i.e., every fuzzy closed set $\tilde{D}$ in $\tilde{B}$, $f^{-1}(\tilde{D})$ is fuzzy θ-closed in $\tilde{A}$.

(e) The inverse image of every fuzzy open set in $\tilde{B}$ is fuzzy θ-open in $\tilde{A}$, i.e., every fuzzy open set $\tilde{D}$ in $\tilde{B}$, $f^{-1}(\tilde{D})$ is a fuzzy θ-open in $\tilde{A}$.

Proof:

(a) ⇒ (b). Let $\mu_{(x_r)} \leq \mu_{\theta cl(\tilde{C})}$ be and let $\tilde{V}$ any open q-nbd. of $f(x_r)$. Then there exists an open q-nbd. $\tilde{U}$ of x_r such that $\mu_{fcl(\tilde{U})} \leq \mu_{(\tilde{V})}$ i.e., f is a fuzzy θ-continuous.

Now $\mu_{(x_r)} \leq \mu_{\theta cl(\tilde{C})} \Rightarrow cl(\tilde{U})\ q\ \tilde{C} \Rightarrow f(cl(\tilde{V}))\ q\ f(\tilde{C}) \Rightarrow \tilde{V}\ q\ f(\tilde{C}) \Rightarrow \mu_{f(x_r)} \leq \mu_{clf(\tilde{C})}$

$\Rightarrow \mu_{(x_r)} \leq \mu_{f^{-1}clf(\tilde{C})}$

Hence, $\mu_{\theta cl(\tilde{C})} \leq \mu_{f^{-1}clf(\tilde{C})}$ and so $\mu_{f(\theta cl(\tilde{C}))} \leq \mu_{clf(\tilde{C})}$

(b) ⇒ (c). Let $\tilde{D}$ be a fuzzy set in $\tilde{B}$.

By (b)) $\mu_{f\theta(clf^{-1}(\tilde{D})} \leq \mu_{clf(f^{-1}(\tilde{D})} \leq \mu_{\theta clf(\tilde{D})}$

$\Rightarrow \mu_{f\theta(clf^{-1}(\tilde{D})} \leq \mu_{cl(\tilde{D})} \Rightarrow \mu_{(f^{-1}f(\theta cl(f^{-1}(\tilde{D}))} \leq \mu_{(f^{-1}cl(\tilde{D})}$

$\Rightarrow \mu_{(\theta cl(f^{-1}(\tilde{D}))} \leq \mu_{(f^{-1}cl(\tilde{D})}$

(c) ⇒ (d). Let $\tilde{D}$ be a fuzzy closed in $\tilde{B}$.

By (c), we have $\mu_{\theta cl(f^{-1}(\tilde{D})} \leq \mu_{f^{-1}cl(\tilde{D})} = \mu_{f^{-1}(\tilde{D})}$

(since $\tilde{B}$ closed), which implies that $\mu_{f^{-1}(\tilde{D})} = \mu_{\theta cl(f^{-1}(\tilde{D})}$

Hence $f^{-1}(\tilde{D})$, is fuzzy θ-closed

(d) ⇒ (e). Let $\tilde{D}$ be a fuzzy open in $\tilde{B}$.

Then $\mu_{(\tilde{A}-\tilde{D})}(X)$ is a fuzzy closed and by (d), $\mu_{f^{-1}(\tilde{A}-\tilde{D})} = \mu_{(\tilde{A}-f^{-1}(\tilde{D})}$ is fuzzy θ-closed.

Hence, $f^{-1}(\tilde{D})$ is fuzzy θ-open.

(e) ⇒ (a) Let x_r be a fuzzy point in $\tilde{A}$ and $\tilde{V}$ be a fuzzy open q-nbd. of $f(x_r)$ by (e) $\mu_{f^{-1}(\tilde{V})}$ is fuzzy θ-open in $\tilde{A}$

Now, $f(x_r)\ q\ x_r \Rightarrow x_r\ q\ f^{-1}(\tilde{V}) \Rightarrow \mu_{(x_r)} \nleq \mu_{\tilde{A}-f^{-1}(\tilde{V})}$

Hence $\mu_{\tilde{A}-f^{-1}(\tilde{V})}(X)$ is fuzzy θ-closed set such that $\mu_{(x_r)} \nleq \mu_{\tilde{A}-f^{-1}(\tilde{V})}$

Then, there exists fuzzy open q-nbd $\tilde{U}$ of x_r, such that $cl(\tilde{U})\ \not{q}\ (\tilde{A} - f^{-1}(\tilde{V}))$, which implies $f(cl(\tilde{U})) \subseteq \tilde{V}$.

This shows that f is fuzzy strong θ-continuous function. ■

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