A study about
β-fuzzy topological group

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Abstract

In this paper we considered fuzzy topological group and β-fuzzy topological group, and we proved some results about the connection between these two concepts.

Keywords: fuzzy topological group, Quotient fuzzy topological group, β-fuzzy topological group

1. Introduction

In his classical paper [L. A. ZADEH] in 1965 Zadeh introduced the notation of fuzzy sets and fuzzy set operation. Subsequently, Chang [C. L.CHANG], applied some basic concepts from general topology to fuzzy sets, he also developed a theory of fuzzy topological spaces. In this paper we study some properties of fuzzy topological groups, and by depending on concepts introduced in [I. M. Hanafy] and [N.R.Das[a,*] ,prabin das[b]] we define the new concept named by β-fuzzy topological group, also we proved some properties on it.

2. Preliminary

2.1 Definition [D. H. Foster]

Let X be a set and I the unit interval [0.1].a fuzzy set $\tilde{A}$ in X is characterized by a membership function $M_{\tilde{A}}$ which is associate with each point $x \in X$ its "grade of membership" $M_{\tilde{A}} \in$ I

2.2 Definition 2.2. [TUNA HATICE YALVAC]

Let f be a mapping from X to a set Y. Let $\widetilde{B}$ be a fuzzy set in Y, with membership function $M_{\widetilde{B}}$. Then the inverse image of $\widetilde{B}$ written $f^{-1}(\widetilde{B})$ is the fuzzy set in X with membership function defined by

$$M_{f^{-1}(\widetilde{B})}(x) = M_{\widetilde{B}}(f(x)) \quad for\,all\; x \in X$$

Conversely, let $\widetilde{A}$ be a fuzzy set in X with membership function $M_{\widetilde{A}}$ then the image of $\widetilde{A}$ written $f(\widetilde{A})$, is the fuzzy set in Y with membership function defined by

$$M_{f(\widetilde{A})}(y) = \begin{cases} \sup\{M_{\widetilde{A}}(x) : x \in X, f(x) = y\} & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in Y$ where $f^{-1}(y) = \{x / \quad f(x) = y\}$

2.3 Theorem [TUNA HATICE YALVAC]

Let f be a function from X to Y and I be any index set. Then the following Statements are true.

1-if $\widetilde{A} \subseteq X$ then $f(\widetilde{A})^C \subseteq f(\widetilde{A}^C)$

2-if $\widetilde{B} \subseteq Y$ then $f^{-1}(\widetilde{B}^C) = f^{-1}(\widetilde{B})^C$

3-if $\widetilde{A}_1, \widetilde{A}_2 \subseteq X$ and $\widetilde{A}_1 \subseteq \widetilde{A}_2$ then $f(\widetilde{A}_1) \subseteq f(\widetilde{A}_2)$

4-if $\widetilde{B}_1, \widetilde{B}_2 \subseteq Y$ and $\widetilde{B}_1 \subseteq \widetilde{B}_2$ then $f^{-1}(\widetilde{B}_1) \subseteq f^{-1}(\widetilde{B}_2)$

5-if $\widetilde{A} \subseteq X$ then $\widetilde{A} \subseteq f^{-1}(f(\widetilde{A}))$

6- if $\widetilde{B} \subseteq X$ then $f(f^{-1}(\widetilde{B})) \subseteq \widetilde{B}$

7-if $\widetilde{A}_i \subseteq X$ $for\,every\,i \in I$ then $f(\bigcup_{i \in I} \widetilde{A}_i) = \bigcup_{i \in I} f(\widetilde{A}_i)$

8-if $\widetilde{B}_i \subseteq Y$ $for\ every\ i \in I$ then $f^{-1}(\bigcup_{i \in I} \widetilde{B}_i) = \bigcup_{i \in I} f^{-1}(\widetilde{B}_i)$

9- if $\widetilde{B}_i \subseteq Y$ $for\ every\ i \in I$ then $f^{-1}(\bigcap_{i \in I} \widetilde{B}_i) = \bigcap_{i \in I} f^{-1}(\widetilde{B}_i)$

10- if $\widetilde{A}_1, \widetilde{A}_2 \subseteq X$ then $f(\widetilde{A}_1 \cap \widetilde{A}_2) \subseteq f(\widetilde{A}_2) \cap f(\widetilde{A}_1)$

11- if f is one to one and $\widetilde{A} \subseteq X$ then $\widetilde{A} = f^{-1}(f(\widetilde{A}))$

12-if f is onto and $\widetilde{B} \subseteq X$ then $f(f^{-1}(\widetilde{B})) = \widetilde{B}$

13-Let g be a function from $Y\, to\, Z$ if $\widetilde{B} \subseteq Z$ then

$(gof)^{-1}(\widetilde{B}) = f^{-1}(g^{-1}(\widetilde{B}))$ and if $\widetilde{A} \subseteq X$ then $(gof)(\widetilde{A}) = g(f(\widetilde{A}))$

14- if f is bisection then for $\widetilde{A} \subseteq X$ then $f(\widetilde{A})^C = f(\widetilde{A}^C)$

2.4 Definition.[R. Lowen]

A fuzzy topology is a family $\widetilde{T}$ of fuzzy sets in X which satisfies the following conditions

1- $k_c \in \widetilde{T} \quad c \in [0,1]$

2- if $\widetilde{A}, \widetilde{B} \in \widetilde{T}$ then $\widetilde{A} \cap \widetilde{B} \in \widetilde{T}$

3-if $\widetilde{A}_i \in \widetilde{T} \quad \forall i \in I$ then $\bigcup_{i \in I} \widetilde{A}_i \in \widetilde{T}$

the pair $(X, \widetilde{T})$ is a fuzzy topological space (FTS).Every member of $\widetilde{T}$ is called a $\widetilde{T}$ -open fuzzy set in $(X, \widetilde{T})$ (or simply open fuzzy set) and complement of a open fuzzy set is called closed fuzzy set

2.5 Definition [N.R.Das[a,*] ,prabin das[b],]

A fuzzy topology $\tilde{T}$ on a group G is said to be compatible if the mapping

$$g: (G \times G, \tilde{T} \times \tilde{T}) \to (G, \tilde{T}),\, g(x,y) = xy$$

and

$$h: (G, \tilde{T}) \to (G, \tilde{T}), h(x) = x^{-1}$$

are fuzzy continuous

2.6 Definition. [N.R.Das[a,*] ,prabin das[b],]

A group G equipped with a compatible fuzzy topology $\tilde{T}$ on G is called a fuzzy topological group (FTG)

2.7 Quotient group

Let G be a group and Hbe a normal subgroup of G for each $x \in G$, Let xH denote the unique coset to which x belongs in the decomposition of G into pair wise disjoint cosets. Let $\varphi: G \to G/H$ define by $\varphi(x) = xH$

It is easy to check φ is homomorphism of G into G/H.

To show this let $x, y \in G$, $\varphi(xy) = xyH = xHyH = \varphi(x)\varphi(y)$ Hence φ is homomorphism.

2.8 Quotient fuzzy topology on $G/\widetilde{H}$ as follows:

A fuzzy set $\widetilde{W}$ in $G/\widetilde{H}$ is open fuzzy if and only if $\varphi^{-1}(\widetilde{W})$ is open fuzzy in G.Note that $\dot{\tilde{A}} = \{a\widetilde{H}; a \in \tilde{A}\}$is open fuzzy if and only if $\cup \{a\widetilde{H};\ a \in \tilde{A}\}$ is an open fuzzy in G

2.9 Theorem [N.R.Das^{a,*} ,prabin das^{b},]

Let $(G, \tilde{T})$ be an FTG and a, b∈ G then

1-The translation maps

$$r_a: (G, \tilde{T}) \to (G, \tilde{T}), r_a(x) = xa$$

and

$$l_a: (G, \tilde{T}) \to (G, \tilde{T}), l_a(x) = ax$$

2-The inversion map

$$f: (G, \tilde{T}) \to (G, \tilde{T}), f(x) = x^{-1}$$

and the map

$$\varphi: (G, \tilde{T}) \to (G, \tilde{T}), \varphi(x) = axb$$

are all fuzzy homeomorphisms.

3. Fuzzy topological group

3.1.Theorem

Let G be a group having fuzzy topology $\tilde{T}$ then $(G, \tilde{T})$ is FTG if and only if the map $g: (G \times G, \tilde{T} \times \tilde{T}) \to (G, \tilde{T})$ define by $g(x, y) = xy^{-1}$ is fuzzy continuous

3.2.Theorem

Let $(G, \tilde{T})$ be FTG, $\tilde{A}, \widetilde{B} \subseteq G,\ g \in G$ then

1- $\tilde{A}$ open fuzzy implies $\tilde{A}g, g\tilde{A}, g\tilde{A}g^{-1}$ and $\tilde{A}^{-1}$ are open fuzzy

2- $\tilde{A}$ closed fuzzy implies $\tilde{A}g, g\tilde{A}, g\tilde{A}g^{-1}$ and $\tilde{A}^{-1}$ are closed fuzzy

3- $\tilde{A}$ open fuzzy implies $\tilde{A}\tilde{B} and \tilde{B}\tilde{A}$ open fuzzy

4- $\tilde{A}$ closed fuzzy and $\tilde{B}$ finite implies $\tilde{A}\tilde{B} and \tilde{B}\tilde{A}$ closed fuzzy .

Proof:

1 and 2 $r_a, l_a\ f, \varphi$ being fuzzy homeomorphism. Are all open fuzzy and closed fuzzy

3-$\tilde{A}\tilde{B} = \cup \{\tilde{A}b; b \in \tilde{B}\}$ is the union of open fuzzy sets and hence open fuzzy similarly for $\tilde{B}\tilde{A}$

4-Similarly as above. ■

3.3.Definition

A FTG G is called fuzzy homogenous if for any $a, b \in G$ there is a fuzzy homeomorphism $f: G \to G\ s.t\ f(a) = b$.

3.4 Theorem

A FTG is a fuzzy homogenous space.

Proof:

Let G be a FTG and $x_1, x_2 \in G$ $take\ \ a = x_1^{-1}x_2$

$f(x) = r_a(x) = xa = xx_1^{-1}x_2$ $implise\ \ f(x_1) = x_2$ by theorem 2.9 f is fuzzy homeomorphism ■

3.5 Theorem

A non trivial FTG has no fixed point properties

Proof:

Let G be a FTG, $a \in G$ with $a \neq e$ now the map $r_a: G \to G$ is fuzzy continuous suppose $r_a(x) = x$ for some $x \in G$ $xa = x$ Implies $a = e$ which is contradiction to the concepts that r_a has no fixed point, hence G has no fixed point properties ■

3.6 Theorem

Every open fuzzy subgroup of FTG is closed fuzzy

Proof:

Let G be a FTG and $\widetilde{H}$ be open fuzzy subgroup of G

$G - \widetilde{H} = \cup\{g\widetilde{H}; g \notin \widetilde{\mathrm{H}}\} = \{r_g(x); g \notin \widetilde{\mathrm{H}}\}$ which is open fuzzy by theorem 3.2 There fore $\widetilde{H}$ is closed fuzzy ■

3.7 Theorem

Let G be FTG and $\widetilde{H}$ be a fuzzy subgroup of G.let $G/\widetilde{H}$ be quotient space, endowed with the quotient topology and φ the canonical mapping of G into $G/\widetilde{H}$

Then

1- φ is onto

2- φ is fuzzy continuous

3- φ is fuzzy open

Proof:

1- Let $p \in G/\widetilde{H}$ implies $p = x\widetilde{H} = \varphi(x)$ we get φ is onto
2- It is clear from definition
3- Let $\tilde{A}$ be any open fuzzy in G we have show that $\varphi(\tilde{A})$ is open fuzzy in $G/\widetilde{H}$ i.e. $\varphi^{-1}(\varphi(\tilde{A}))$ is open fuzzy in G.

$$\varphi^{-1}\left(\varphi(\tilde{A})\right) = \{x; x \in a\widetilde{H}\ for\ some\ a \in \tilde{A}\} = \tilde{A}\widetilde{H}$$

By theorem 3.2 $\widetilde{H}\tilde{A}$ is open fuzzy, therefore f is fuzzy open mapping ■

3.8.Theorem

Let G be FTG and $\widetilde{H}$ be fuzzy subgroup of G then $G/\widetilde{H}$ is fuzzy discrete topological space if and only if $\widetilde{H}$ is open fuzzy

Proof:

Let $G/\widetilde{H}$ be a fuzzy discrete topological space then each subset of $G/\widetilde{H}$ is open fuzzy and hence each singleton is open fuzzy in particular $\widetilde{H}$ is fuzzy point of $G/\widetilde{H}$, there for $\widetilde{H}$ is open fuzzy

Conversely, let $\widetilde{H}$ be open fuzzy so is $x\widetilde{H}$ open fuzzy for each $x \in G$ by theorem 3.2 This show that each singleton $\{\dot{x}\}$ is open fuzzy in $G/\widetilde{H}$, Therefore $G/\widetilde{H}$ is fuzzy discrete space ■

3.9 Theorem

Let G be FTG and $\widetilde{H}$ normal fuzzy subgroup of G and $G/\widetilde{H}$ endowed with the quotient fuzzy topology, is FTG.

Proof:

Let G be FTG to prove $(\dot{x},\dot{y}) \to \dot{x}\dot{y}^{-1}$ of $G/\widetilde{H} \times G/\widetilde{H}$ *onto* $G/\widetilde{H}$ is fuzzy continuous. Let $\widetilde{W}$ be any open fuzzy of $\dot{x}\dot{y}^{-1}$ where $\dot{x} = x\widetilde{H}$ *and* $\dot{y} = y\widetilde{H}$ clearly $\varphi^{-1}(\widetilde{W})$ is open fuzzy in G and $xy^{-1} \in \varphi^{-1}(\widetilde{W})$, since G is FTG $\exists\ \widetilde{U},\widetilde{V}$ *in* G such that $x \in \widetilde{U}, y \in \widetilde{V}$ and $xy^{-1} \in \widetilde{U}\widetilde{V}^{-1} \subseteq \varphi^{-1}(\widetilde{W})$ $\dot{x}\dot{y}^{-1} \in \varphi(\widetilde{U})\varphi^{-1}(\widetilde{V}) \subseteq \varphi\left(\varphi^{-1}\left(\widetilde{W}\right)\right) = \widetilde{W}$ by theorem 3.7 $\varphi$is open fuzzy mapping we get there exist an open sets $\varphi(\widetilde{U})$ and $\varphi(\widetilde{V}^{-1})$ of $\dot{x}$ *and* $\dot{y}^{-1}$ respectively there fore $G/\widetilde{H}$ is FTG ■

4. β - fuzzy topological group

4.1 Definition

Let G be a group and $(G,\widetilde{T})$ be a fuzzy topological space. $(G,\widetilde{T})$ is called β - fuzzy topological group or β-FTG for short if

the maps $g:(G,\widetilde{T})\times(G,\widetilde{T})\to(G,\widetilde{T})$ defined by $g(x,y)=xy$ and $h:(G,\widetilde{T})\to(G,\widetilde{T})$ defined by $h(x)=x^{-1}$ are fuzzy $M\beta$-continuous

4.2 Theorem

Let G be group having fuzzy topology. $(G,\widetilde{T})$ is β-FTG if and only if the mapping $g:(G,\widetilde{T})\times(G,\widetilde{T})\to(G,\widetilde{T})$ is defined by $g(x,y)=xy^{-1}$ is fuzzy $M\beta$-continuous.

4.3 Theorem

Let a be a fixed element of β-fuzzy topological group $(G,\tilde{T})$ then the mappings

$r_a(x) = xa, l_a(x) = ax,\ f(x) = x^{-1}\ and\ g(x) = axa^{-1}$ of $(G,\tilde{T})$ onto $(G,\tilde{T})$ are fuzzy $M\beta$-homeomorphisms of G

Proof:

It is clear r_a is one to one and onto

Let $\tilde{W}$ be any β -open fuzzy set containing xa since $(G,\tilde{T})$ is β-FTG there exist β-open fuzzy set $\tilde{U}\ of\ x \quad s.t\ \tilde{U}a \subseteq \tilde{W}$ we get $r_a(\tilde{U}) \subseteq \tilde{W}$

Hence r_a is fuzzy $M\beta$-continuous, It is easy to see that r_a^{-1} of r_a is the mapping $r_a^{-1}(x) = xa^{-1}$ which is fuzzy $M\beta$-continuous by the same way of r_a ,Therefore r_a is $M\beta$-homeomorphism .Similarly for l_a

Finally the composition of two fuzzy $M\beta$-homeomorphisms

$r^{-1}{}_a(x) = xa^{-1}\ and\ l_a(x) = ax$ is fuzzy $M\beta$-homeomorphisms

Hence $g(x) = r_a^{-1} \circ l_a(x) = axa^{-1}$ is fuzzy $M\beta$-homeomorphisms ■

4.4 Corollary

Let $\tilde{F}$ be β-closed fuzzy, $\tilde{P}$ be an β-open fuzzy and $\tilde{A}$ any subset of β-FTG $(G,\tilde{T})$ and $a \in G$ then

1- $\tilde{F}a, a\tilde{F}$ and $\tilde{F}^{-1}$ are β-closed fuzzy

2- $\tilde{P}a\ , a\tilde{P}, \tilde{A}\tilde{P}, \tilde{P}\tilde{A}\ and\ \tilde{P}^{-1}$ are β-open fuzzy

4.5 Corollary

Let $(G,\tilde{T})$ be β-FTG for any $x_1, x_2 \in G$ there exists a fuzzy $M\beta$-homeomorphism $f\ of\ G\ \ s.t\ f(x_1) = x_2$

Proof:

Let $x_1^{-1}x_2 = a \in G$ and consider the mapping $f : (G,\tilde{T}) \to (G,\tilde{T})$ defined by $f(x) = xa$ then f is fuzzy $M\beta$-homeomorphism by theorem 4.3 $f(x_1) = x_2$ ■

A space for which corollary 4.5 is true is called a fuzzyβ-homogeneous space

4.6 Theorem

Let $(G,\widetilde{T})$ be β-FTG and $\widetilde{A},\widetilde{H}$ are fuzzy subset of G then

1- $\beta cl(a\widetilde{A}a^{-1}) = a\beta cl(\widetilde{A})a^{-1}$ where $a \in G$ is definite point

2- if $\beta cl(\widetilde{A}) \times \beta cl(\widetilde{H}) \subseteq \beta cl(\widetilde{A} \times \widetilde{H}), \beta cl(\widetilde{A})\beta cl(\widetilde{H}) \subseteq \beta cl(\widetilde{A}\widetilde{H})\, and$

$\beta cl(\widetilde{A})\beta cl(\widetilde{H}^{-1}) \subseteq \beta cl(\widetilde{A}\widetilde{H}^{-1})$

Proof:

By corollary 4.4 $a\beta cl(\widetilde{A})a^{-1}$ is β-closed fuzzy set, since is the smallest β-closed fuzzy set containing $a\widetilde{A}a^{-1}$, $\beta cl(a\widetilde{A}a^{-1}) \subseteq a\beta cl(\widetilde{A})a^{-1}$

Let $f:(G,\widetilde{T}) \to (G,\widetilde{T})$ be a map defined by $f(x) = axa^{-1}$

by theorem 4.3 f is fuzzy $M\beta$-homeomorphism by lemma2.5in paper [I. M. Hanafy]

$f(\beta cl(\widetilde{A})) \subseteq \beta cl(f(\widetilde{A}))$ thus

$a\beta cl(\widetilde{A})a^{-1} \subseteq \beta cl(a\widetilde{A}a^{-1})\; we\; get\; a\beta cl(\widetilde{A})a^{-1} = \beta cl(a\widetilde{A}a^{-1})$

For proof part 2 by theorem 4.6 the map $g:(G,\widetilde{T}) \times (G,\widetilde{T}) \to (G,\widetilde{T})$ is defined by

$g(x,y) = xy^{-1}$ is fuzzy $M\beta$-continuous, since $\beta cl(\widetilde{A}) \times \beta cl(\widetilde{H}) \subseteq \beta cl(\widetilde{A} \times \widetilde{H})$,

$f(\beta cl(\widetilde{A}), \beta cl(\widetilde{H})) \subseteq f(\beta cl(\widetilde{A} \times \widetilde{H}))$

Since f is fuzzy $M\beta$-continuous $f(\beta cl(\widetilde{A} \times \widetilde{H})) \subseteq \beta cl f(\widetilde{A},\widetilde{H}))$ By lemma 2.5 in paper [I. M. Hanafy] thus $\beta cl(\widetilde{A})\beta cl(\widetilde{H})^{-1} \subseteq \beta cl(\widetilde{A}\widetilde{H}^{-1})$,for $x \in G$

$$M_{\beta cl(\widetilde{H}^{-1})}(x) = M_{\cap\{\widetilde{K}_i : \widetilde{H}^{-1} \subseteq \widetilde{K}_i, \widetilde{K}_i \text{ is } \beta-closed\}}(x) = \inf\{M_{\widetilde{k}_I}(x) : \widetilde{H}^{-1} \subseteq \widetilde{K}_i\}$$

$$= \inf\{M_{\widetilde{K}_i^{-1}}(x^{-1}) : \widetilde{H} \subseteq \widetilde{K}_i^{-1}\} = M_{\cap\{\widetilde{K}^{-1}{}_i : \widetilde{H} \subseteq \widetilde{K}_i^{-1}\}}(x^{-1}) = M_{\beta cl(\widetilde{H})}(x^{-1}) = M_{\beta cl(\widetilde{H})^{-1}}(x)$$

We get $\beta cl(\widetilde{H}^{-1}) = \beta cl(\widetilde{H})^{-1}$ hence $\beta cl(\widetilde{A})\beta cl(\widetilde{H}^{-1}) \subseteq \beta cl(\widetilde{A}\widetilde{H}^{-1})$

Similarly for $\beta cl(\widetilde{A})\beta cl(\widetilde{H}) \subseteq \beta cl(\widetilde{A}\widetilde{H})$ ■

4.7 Theorem

Let $(G,\widetilde{T})$ be an β-FTG and $\beta cl(\widetilde{A}) \times \beta cl(\widetilde{H}) \subseteq \beta cl(\widetilde{A} \times \widetilde{H})$ if $\widetilde{H}$ is a fuzzy subgroup of G then $\beta cl(\widetilde{H})$ is also fuzzy subgroup of G, if $\widetilde{H}$ is a fuzzy normal subgroup of G then $\beta cl(\widetilde{H})$ is also fuzzy normal subgroup of G

Proof:

In [10] $\tilde{H}\tilde{H} \subseteq \tilde{H} \Rightarrow \beta cl(\tilde{H}\tilde{H}) \subseteq \beta cl(\tilde{H})$ by above theorem

$\beta cl(\tilde{H})\beta cl(\tilde{H}) \subseteq \beta cl(\tilde{H}\tilde{H})$ we get $\beta cl(\tilde{H})\beta cl(\tilde{H}) \subseteq \beta cl(\tilde{H})$ …(1)

Since $\tilde{H}$ is fuzzy subgroup $M_{\tilde{H}}(x) = M_{\tilde{H}}(x^{-1}) = M_{\tilde{H}^{-1}}(x)$ for all $x \in G$

$\tilde{H} = \tilde{H}^{-1}$ and hence $\beta cl(\tilde{H}) = \beta cl(\tilde{H}^{-1})$ we may show that for every $x \in G$

$M_{\beta cl(\tilde{H}^{-1})}(x) = M_{\beta cl(\tilde{H})^{-1}}(x)$ using the same method a above

$M_{\beta cl(\tilde{H})}(x) = M_{\beta cl(\tilde{H})^{-1}}(x) = M_{\beta cl(\tilde{H})}(x^{-1})$ …(2) from (1)&(2)

and in [NASEEM AJMAL]

$\beta cl(\tilde{H})$ is fuzzy subgroup of G ,Let $\tilde{H}$ be a fuzzy normal subgroup of G then

$M_{\tilde{H}}(ab) = M_{\tilde{H}}(ba)$ *for any* $a,b \in G$ and hence

$M_{x\tilde{H}x^{-1}}(z) = M_{x\tilde{H}}(zx) = M_{\tilde{H}}(x^{-1}zx) = M_{\tilde{H}}(x^{-1}xz) = M_{\tilde{H}}(z)$

Hence $x\tilde{H}x^{-1} = \tilde{H}$ *we get* $\beta cl(x\tilde{H}x^{-1}) = \beta cl(\tilde{H})$ by theorem 4.6 hence

$x\beta cl(\tilde{H})x^{-1} = \beta cl(\tilde{H})$ *for every* $x \in G$

$M_{\beta cl(\tilde{H})}(xy) = M_{x\beta cl(\tilde{H})x^{-1}}(xy) = M_{\beta cl(\tilde{H})x^{-1}}(x^{-1}xy) = M_{\beta cl(\tilde{H})x^{-1}}(y) = M_{\beta cl(\tilde{H})}(yx)$

We get $\beta cl(\tilde{H})$ is fuzzy normal subgroup of G ■

4.8 Theorem

Every β-open fuzzy subgroup $\tilde{H}$ of β-FTG $(G,\tilde{T})$ is β-closed fuzzy

Proof:

for each $x \in G, x\tilde{H}$ is β-open fuzzy by corollary (4.4) hence $\tilde{H} = (\bigcup x\tilde{H})^C$ is β-closed fuzzy because $\bigcup x\tilde{H}$ is β-open fuzzy , where the union is taken over all pair wise fuzzy cosets different from $\tilde{H}$ ■

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