A generalized mathematical model for an ac chopper output voltage

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Abstract. AC choppers are modified types of ac voltage controllers. They become very popular nowadays in many applications, because objectionable harmonic components of its output voltage and current can be eliminated. The output voltage waveform of the ac chopper may have many patterns with as many segments as desired. This paper presents a generalized mathematical model for the output voltage waveform of an ac chopper. The output voltage waveform is realized by multiplying the control pulses by a sine wave. Many patterns of the output waveform can be produced by setting suitable constraints. The practical circuit of the proposed model is out of the scope of this paper. The model is implemented using MATLAB. Typical patterns are presented to illustrate the versatility of the proposed model.

Keywords: AC choppers, ac voltage controllers, modeling.

1 INTRODUCTION

AC choppers are devices which convert fixed alternating voltage directly to variable alternating voltage of the same frequency. They can be used in many domestic and industrial applications like automatic voltage regulators (Kwon et al., 2002), (Hietpas et al, 2000) and (Veszpremi et al., 2000), soft-starter and speed regulator of the induction motor (Bodur et al., 2000), (Sundareswaran et al., 2006) and (Fujikura et al., 2007), light dimmer (Geraldo et al., 2007) etc. Usually, insulated gate bipolar transistors (IGBT) are used in ac chopper circuits (Hunyár et al., 2001), (Ursaru et al., 2004) and (Nan et al., 2010).

The output voltage waveform of the ac chopper may have segments of different numbers and positions. Many patterns have been recorded in the literature (Altintas, 2010).

This paper is a modest attempt to develop a generalized mathematical equation for the ac chopper output patterns. Many patterns can be produced by setting suitable constraints. This equation is developed utilizing MATLAB programming.

2 THE DEVELOPED EQUATION

Using MATLAB program, the proposed equation is devised as

```
v=(c1*symsum(heaviside(t-(T/2)*(2*n/k))-heaviside(t-(T/2)*((2*n+1)/k)),n,k1,k2)
+ c2*symsum(heaviside(-t-(T/2)*(2*n/k)+T/2)-heaviside(-t-(T/2)*((2*n+1)/k) +T/2),n,k1,k2)
+ c3*symsum(heaviside(t-(T/2)*(2*n/k)-T/2)-heaviside(t-(T/2) *((2*n+1)/k)-T/2),n,k1,k2)+
c4*symsum(heaviside(-t-(T/2)*(2*n/k)+T)
-heaviside(-t-(T/2)*((2*n+1)/k)+T),n,k1,k2))* sin(2*pi*f*t);
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Where
c1, c2, c3 & c4 are the controlling variables responsible for the existing of each quarter (portion) of the output wave. These variables can be assigned as 0 or 1. When any of them is zero, the corresponding quarter wave will disappear. Putting all these variables as zeros, results in zero output.

k is the total no of segments, which is also responsible for the pulse width, the more the value of k the less the pulse width is.
n is the variable of the series.
k1 is the lower limit of the summation which determines the order of the initial segment in the positive & the negative half cycle. k1 = 0 means we have the first pulse or segment.
k2 is the upper limit of the summation which determines the order of the final segment in the positive & the negative half cycle. k2 = 7 means we have the eighth pulse or segment.
f is the supply frequency in hertz.
T is the period of the waveform corresponding to the supply frequency.
Symsum and Heaviside are the summation notation and the step function, respectively, as defined by MATLAB.

3 VOLTAGE WAVEFORMS

Improvement of the output voltage waveforms was always a main goal for many researchers (Balasubramonian1 et al., 2014) and (Saravanakumar et al., 2015). The above equation is developed so as to obtain a perfect quarter wave and half wave symmetry as follows:

The first quarter of the ac chopper output waveform is obtained by multiplying a train of pulses by a sine function. The train of pulses is obtained by the symsum function. This can be achieved by setting the constants c1 =1 and c2 = c3 = c4 = 0 in the generalized equation. Thus the remaining portion of the equation, "c1*symsum(heaviside(t-(T/2) *(2*n/k))-heaviside(t-(T/2)*((2*n+1)/k)),n,k1,k2)*sin(2*pi*f*t)" gives the first quarter wave.

For 1 p.u and 50 Hz, the train of pulses and the resulting chopped quarter wave is presented in Fig. 1 (a) to (d).

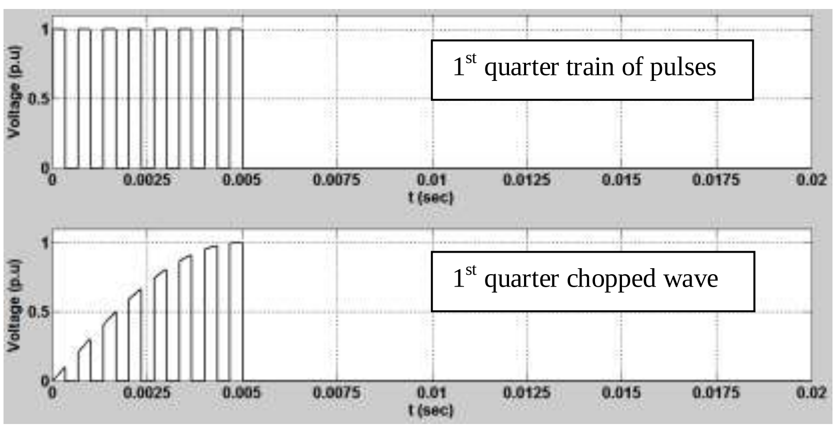

Fig. 1 (a) Developing of first quarter chopped wave

In a similar manner the second, third and fourth quarters can be obtained by setting only c2 = 1, c3 = 1 and c4 = 1 respectively. The resulted waveforms are given in Figs. 1 (b) to (d).

Adding the four quarters, as in the generalized equation, results in the complete chopped wave as shown in Fig. 2.

In Fig. 1 and 2, k is taken as 30 indicating that there are 30 segments throughout the waveform, k1 is taken as 0 which means that the first segment is available, and k2 as 7 indicating that there are 8 segments in each quarter wave (from 0 to 7). It may be noted that the 8th segment of the first quarter and the 1st segment of the second quarter are combined together to form the middle segment of the positive half wave. The total number of the segments in the positive half wave is thus 15. The negative half cycle is formed in a similar manner. Total number of segments can be varied by varying k. However, k must be even to maintain half wave symmetry.

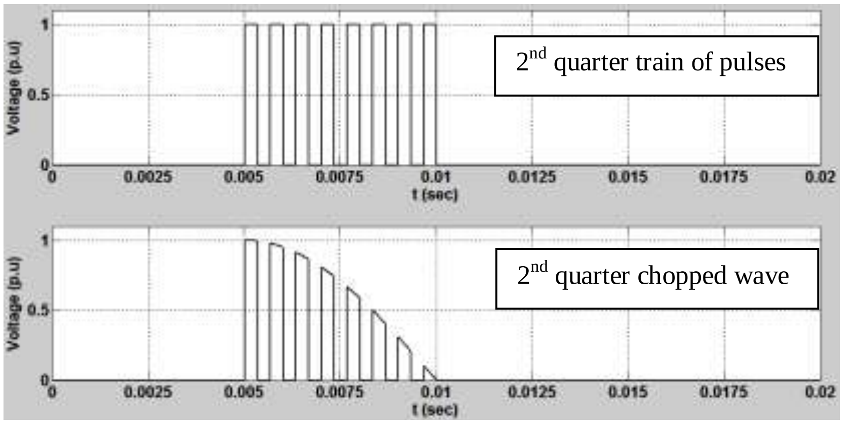

Fig. 1 (b) Developing of second quarter chopped wave

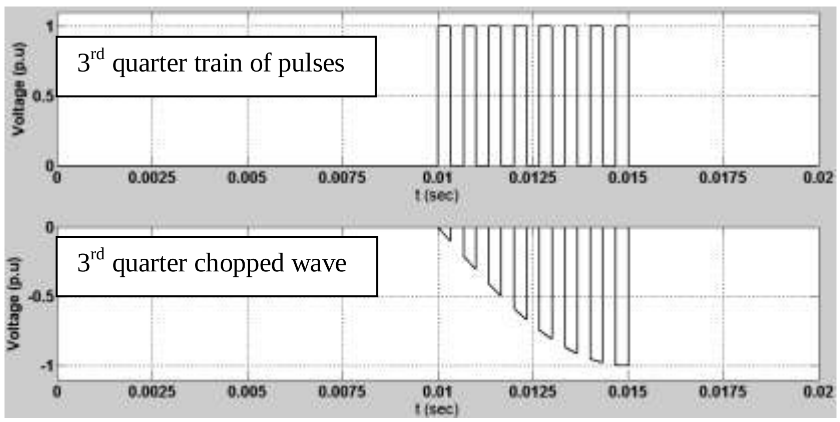

Fig. 1 (c) Developing of third quarter chopped wave

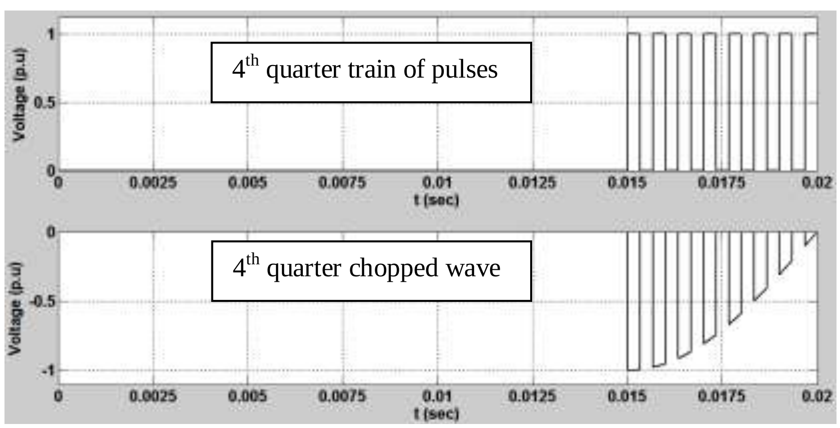

Fig. 1 (d) Developing of fourth quarter chopped wave

Fig. 1 Development of the four quarter waves from the generalized equation

In fact each portion of the proposed equation produces a corresponding portion of a wave. For the above settings, each portion constitutes exactly a quarter wave. If k2 is changed, each resulting portion may be more or less than a quarter wave. However, irrespective of the value of k2, combination of the two positive portions yields the positive half cycles and the same is true for the negative half cycle. Thus, we will keep using the term quarter even if it spans more or less than 90 degrees of a cycle.

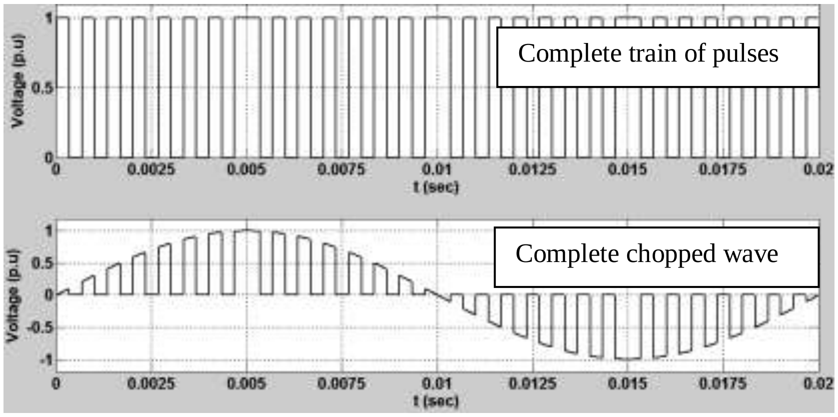

Fig. 2 Development of a full cycle chopped wave

For the above conditions, the output voltage is reproduced in Fig. 3 (a). Combination of the two middle segments in each half cycle is illustrated in Fig. 3 (b).

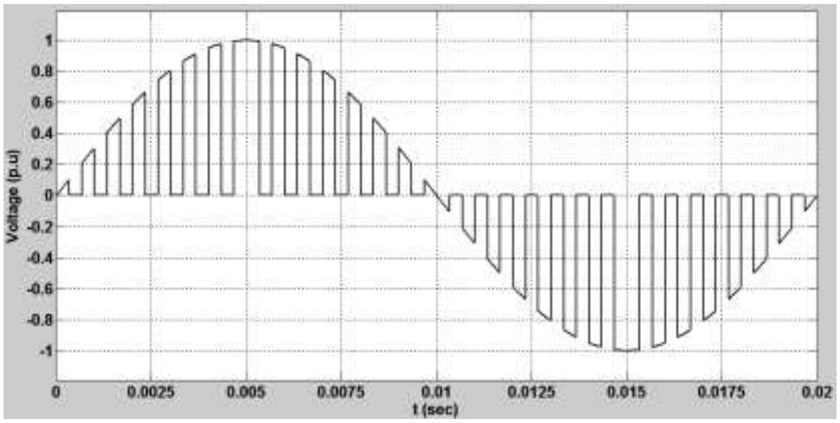

Fig. 3 (a) Output voltage

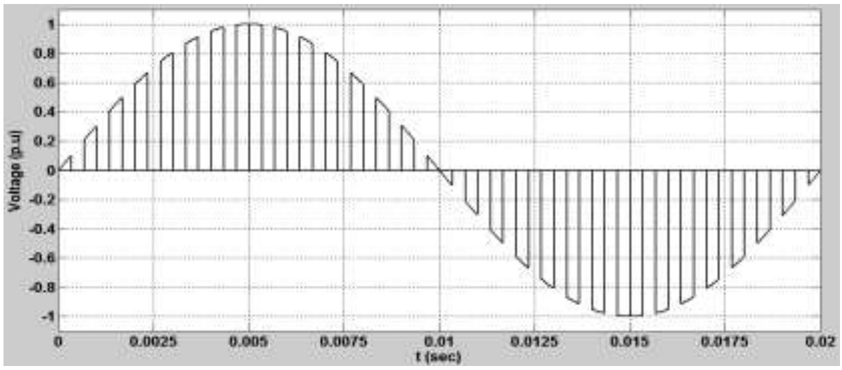

Fig. 3 (b) Combination of the two middle segments in each half cycle

Fig. 3 Construction of the output voltage for k = 30, k1 = 0 and k2 = 7

4 VARYING THE OUTPUT VOLTAGE

The output voltage can be increased or decreased by adjusting k1 or k2.

4.1 Increasing the output voltage

Increasing the output voltage can be made in many ways. One way is by increasing the width of the middle segment. This can be achieved by increasing k2 keeping k1 = 0. For k2 = 8, as an example, the output voltage is presented in Fig. 4 (a). Combination of the middle segments in each half cycle is illustrated in Fig. 4 (b) and (c).

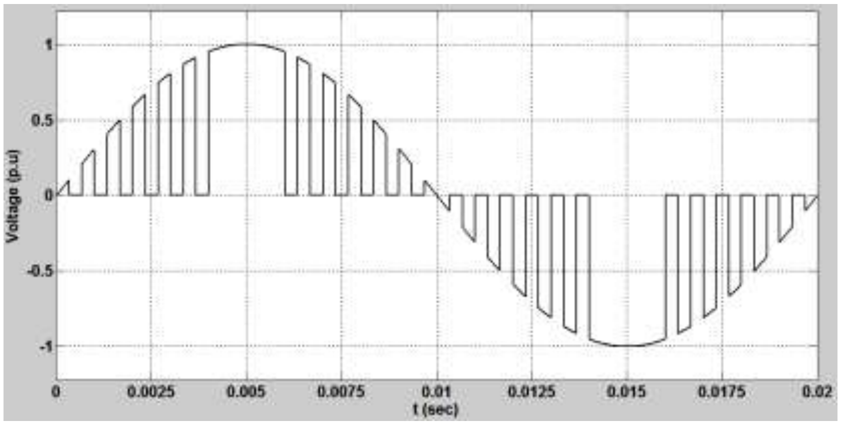

Fig. 4 (a) Output voltage

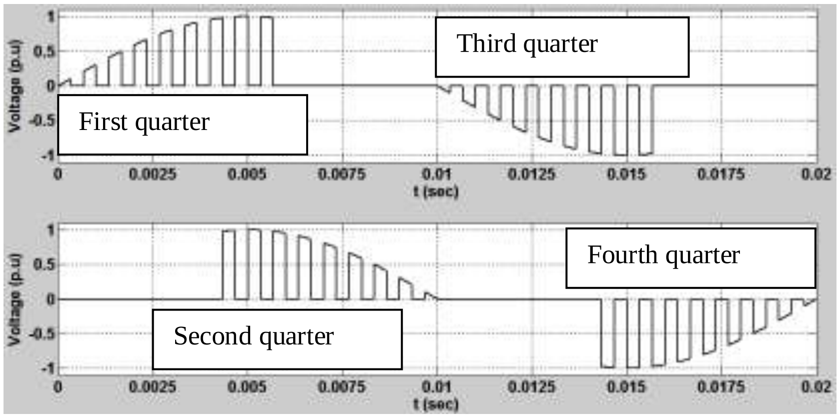

Fig. 4 (b) The four quarters of the output waveform

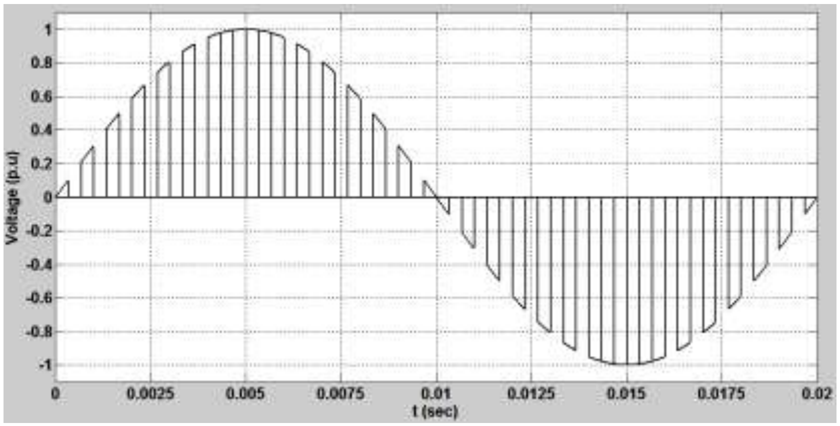

Fig. 4 (c) Combination of the middle segments in each half cycle

Fig. 4 Construction of the output voltage for k = 30, k1 = 0 and k2 = 8

As it is evident from Fig. 4 (b) and (c), six segments constitute the middle segments of positive and negative half cycles.

If k2 is further increased, the width of the middle segment increases. For k2 = 12, the output voltage is shown in Fig. 5.

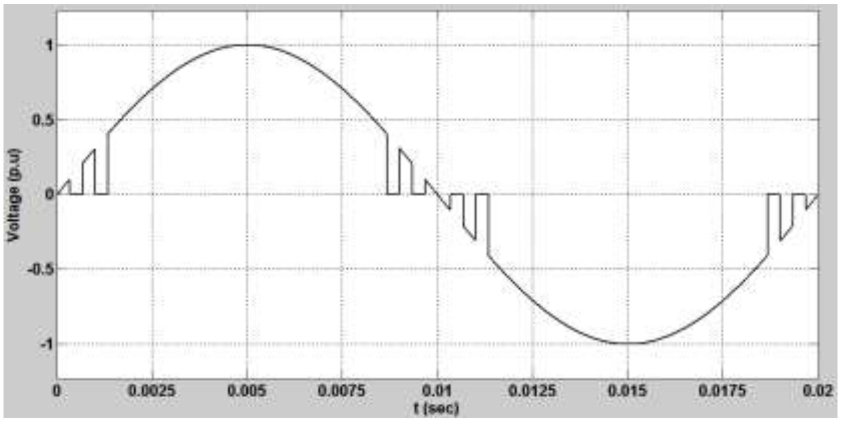

Fig. 5 Output voltage for k = 30, k1 = 0 and k2 = 12

To produce a sine wave, k2 is made 14. The output voltage for such a case is presented in Fig. 6 (a). Construction of this wave is achieved by combining all the segments in the first and second quarters to produce the positive half cycle and the third and fourth quarters to produce the negative half cycle. This is illustrated in Fig. 6 (b) and (c).

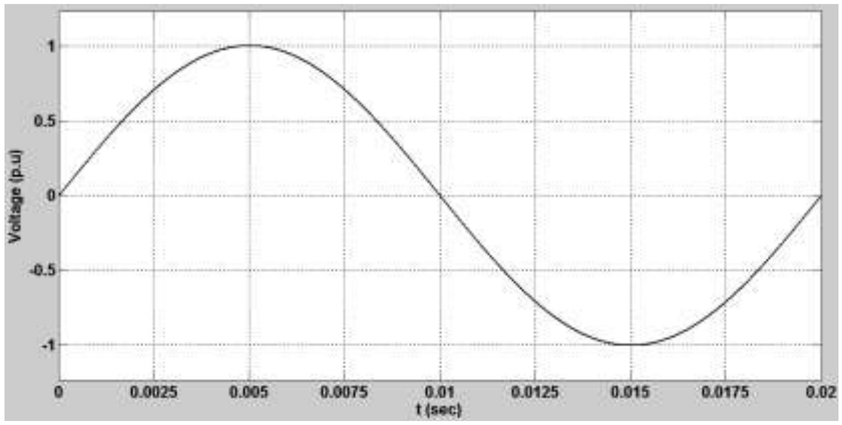

Fig. 6 (a) Output voltage

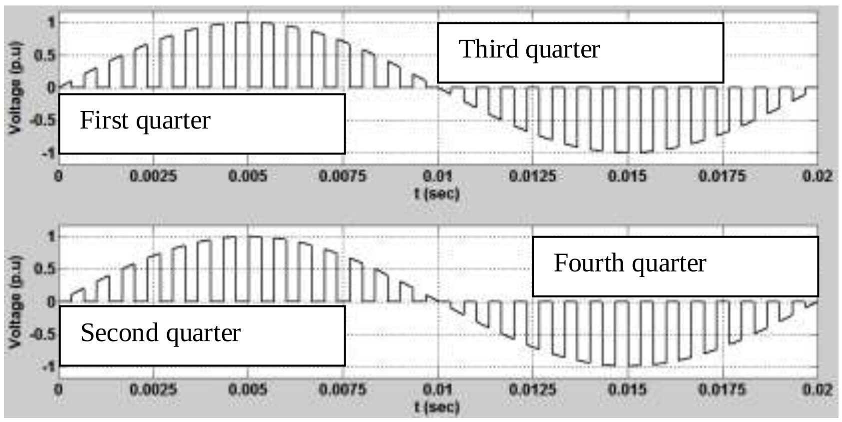

Fig. 6 (b) The four quarters of the output waveform

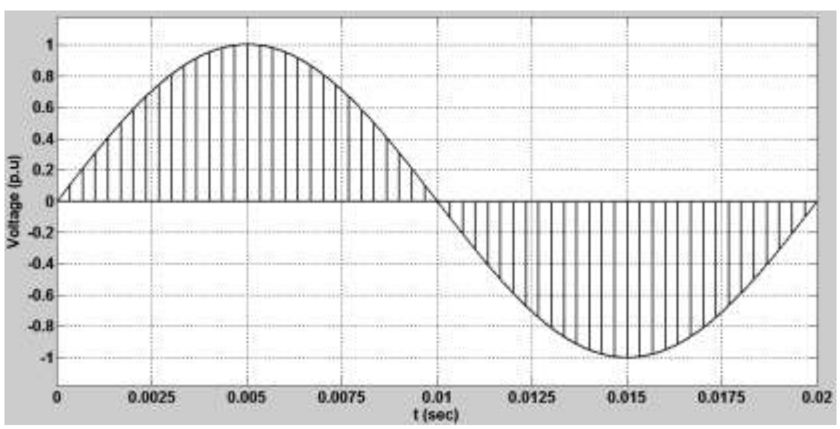

Fig. 6 (c) Combination of all the segments in each half cycle

Fig. 6 Construction of the output voltage for k = 30, k1 = 0 and k2 = 14

4.2 Decreasing the output voltage

Conversely the output voltage can be reduced by increasing k1 or decreasing k2 or varying both of them. For k1 = 2 and k2 = 7, the first segment appears is the third one (from zero to two). The output voltage for such a case is presented in Fig. 7.

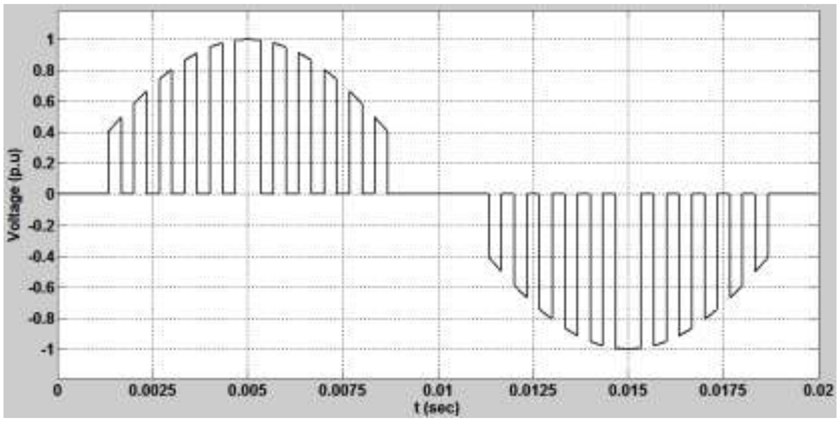

Fig. 7 Output voltage for k = 30, K1=2 and K2=7

Increasing k1 further will reduce the output voltage more and more. Fig. 8 present the output voltage for k1 = 6.

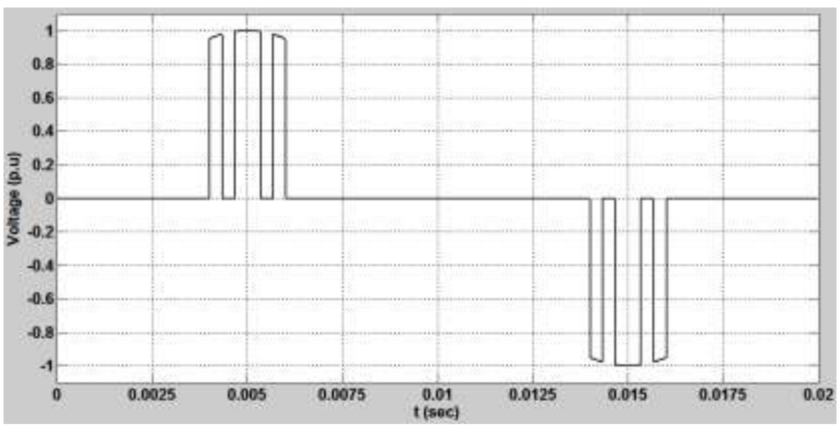

Fig. 8 Output voltage for k = 30, K1 = 6 and k2 = 7

Removing the middle segments, results also in a reduction of the output voltage. This is achieved by reducing k2. For k1 = 0 and k2 = 3, the output voltage is shown in Fig. 9. These constraints mean that there are only four segments at the beginning and four segments at the end of each half wave.

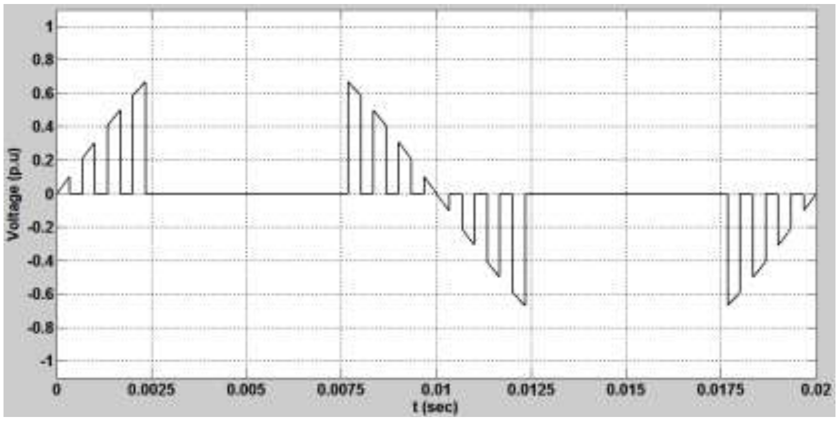

Fig. 9 Output voltage for k = 30, K1 = 0 and k2 = 3

And with k1 = 2 and k2 = 5, the resulting wave appears as in Fig. 10.

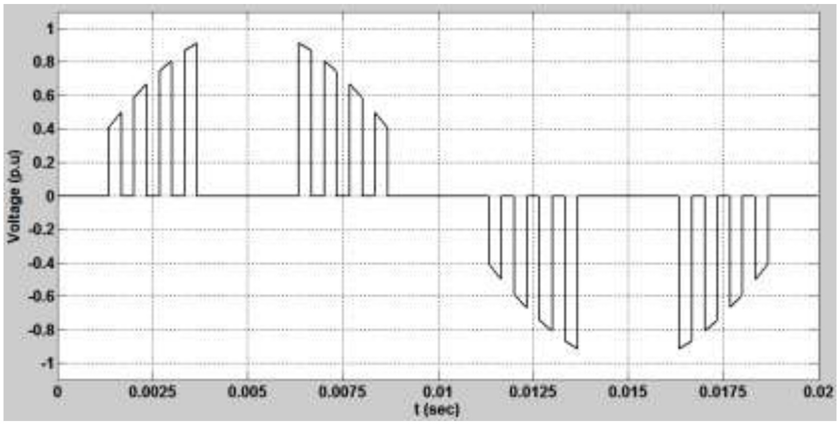

Fig. 10 Output voltage for k = 30, K1 = 2 and k2 = 5

Improving the chopped voltage can be made by increasing the value of k as illustrated in Fig. 11.

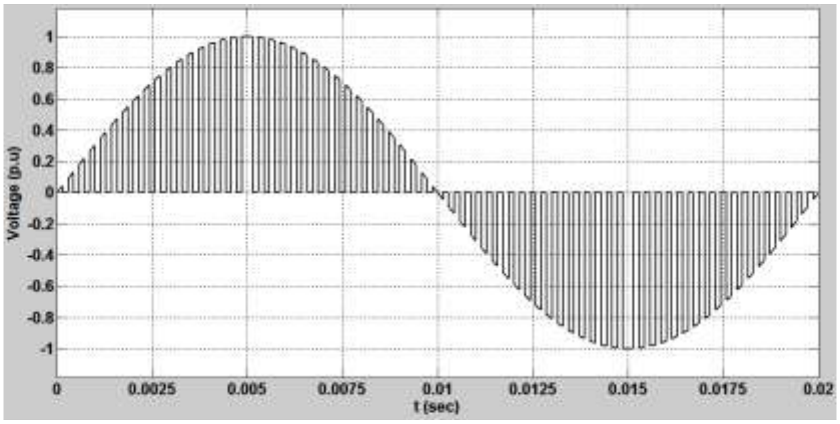

Fig. 11 Output voltage for k = 70, K1 = 0 and k2 = 17

More patterns can also be obtained by just replacing k1 and k2 in c2 and c4 parts of the general equation by k3 and k4 respectively. The modified equation will be

```
v=(c1*symsum(heaviside(t-(T/2)*(2*n/k))-heaviside(t-(T/2)*((2*n+1)/k)),n,k1,k2)
+ c2*symsum(heaviside(-t-(T/2)*(2*n/k)+T/2)-heaviside(-t-(T/2)*((2*n+1)/k)+T/2),n,k3,k4)
+ c3*symsum(heaviside(t-(T/2)*(2*n/k)-T/2)-heaviside(t-(T/2)*((2*n+1)/k)-T/2),n,k1,k2)
+ c4*symsum(heaviside(-t-(T/2)*(2*n/k)+T)-heaviside(-t-(T/2)*((2*n+1)/k)+T),n,k3,k4))
* sin(2*pi*f*t);
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It is noted that by introducing k3 and k4, the quarter wave symmetry gets lost. Typical examples for such patterns are presented in Figs. 12 (a) to (c).

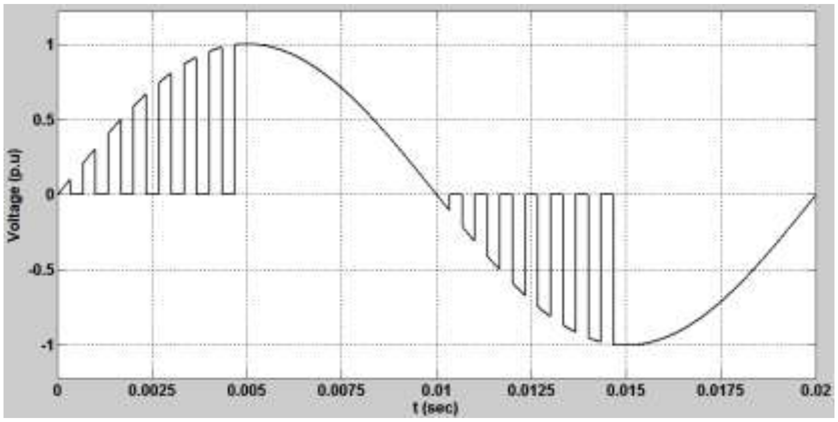

Fig. 12 (a) Output voltage for k = 30, K1 = 0 and k2 = 14, k3 = 0 and k4 = 7

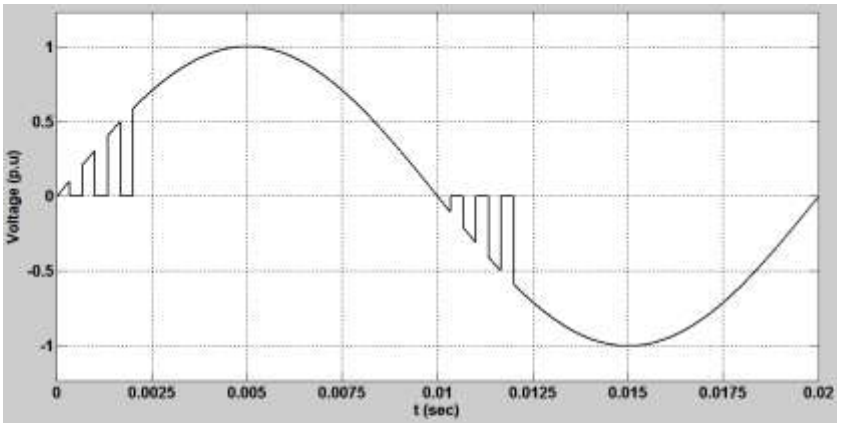

Fig. 12 (b) Output voltage for k = 30, K1 = 0 and k2 = 14, k3 = 0 and k4 = 11

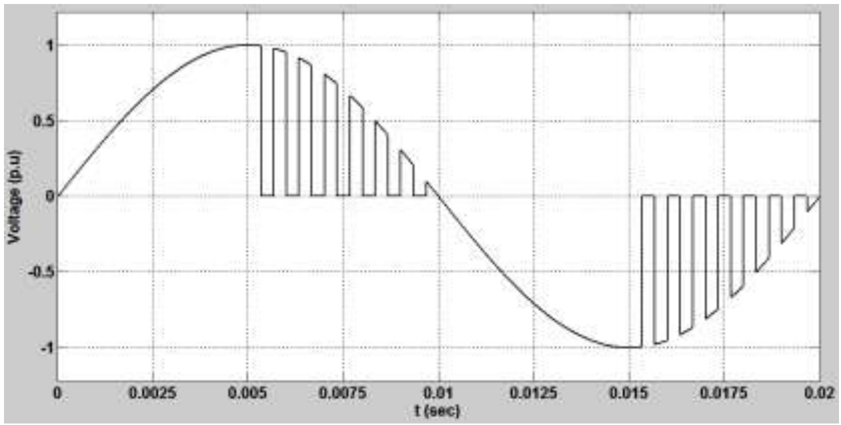

Fig. 12 (c) Output voltage for k = 30, K1 = 0 and k2 = 7, k3 = 0 and k4 = 14

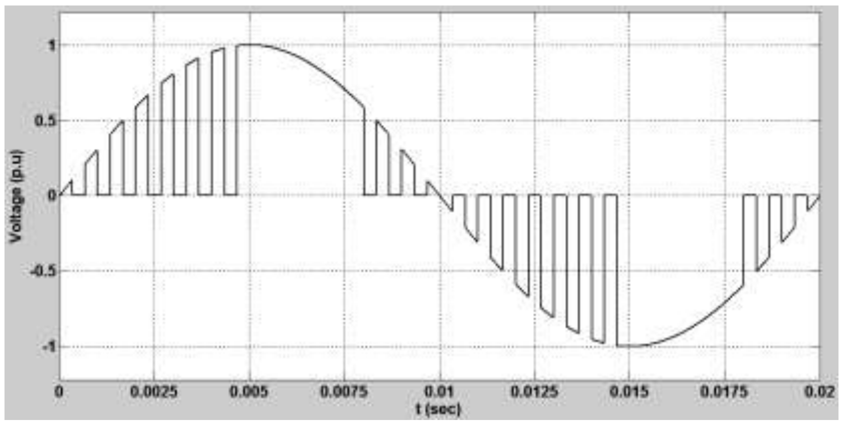

Fig. 12 (d) Output voltage for k = 30, K1 = 0 and k2 = 11, k3 = 0 and k4 = 7

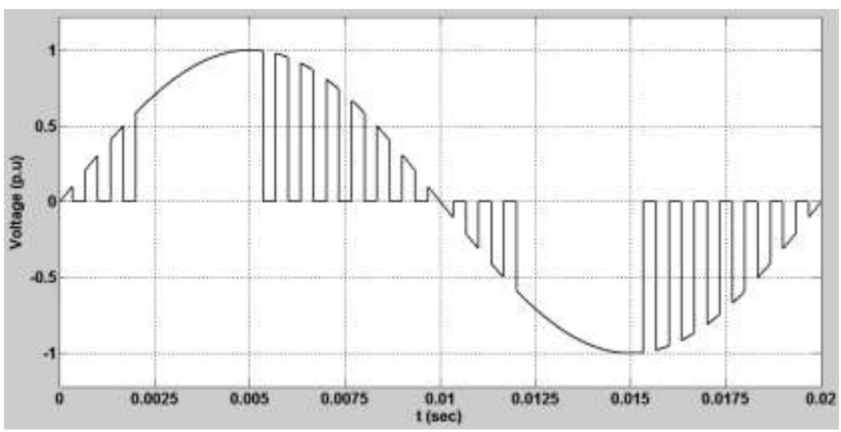

Fig. 12 (e) Output voltage for k = 30, K1 = 0 and k2 = 7, k3 = 0 and k4 = 11

Fig. 12 Miscellaneous patterns with unsymmetrical quarter waves

As seen, many patterns can be produced, however, harmonic content is minimized by making quarter and have wave symmetry.

Although all the previous results have been produced for f = 50 Hz, the above equation holds true for any operating frequency.

5 CONCLUSION

The output voltage waveform of an ac chopper may have many patterns with as many segments as desired.

This paper presents a generalized mathematical model for the control pulses of an ac chopper. The model is developed using MATLAB programming.

Different patterns of the output voltage including sinusoidal waveform can be obtained by varying just two constraints, in the model, to get an output voltage of quarter wave and half wave symmetry. More patterns with quarter wave asymmetry can be obtained by adding two more constraints. Typical patterns are presented to illustrate the versatility of the proposed model.

The output voltage can be controlled by adjusting the model constraints of the generalized equation

Improving the chopped voltage can be made by increasing the number of segment of the voltage waveform. The developed model is capable of producing as many segments as required.

Although ac choppers are used in low frequency power circuits, the developed model is valid for any other operating frequency.

Implementing this model practically minimizes the harmonic content of the output voltage.

References

Altintas, A. (2010). A new method for power quality improvement in classical AC/AC voltage controllers using PWM technique. Scientific Research and Essays Vol. 5(10), pp. 1075-1083.

Balasubramonian, M. & Dharani, S. (2014). Design and Implementation of SHE PWM in a Single Phase ac Chopper Using Generalized Hopfield Neural Network. International Conference on Engineering Technology and Science-(ICETS’14), Volume 3, Special Issue 1, India

Bodur H., Bakan A. & Sarul M. (2000). Universal motor speed control with current controlled PWM AC chopper by using a microcontroller. Proceedings of IEEE International Conference on Industrial Technology, Vol.2, pp. 394-398.

Fujikura, S., Ueda, A. & Torii, A, (2007). Analysis of a Three-Phase Buck-Boost AC Chopper Controlled in Two Phases, Power Conversion Conference, pp.824-830.

Geraldo, C., Sincero, R. & Perin, A. (2007). High Pressure Sodium Lamp High Power Factor Electronic Ballasts Using AC–AC Converters. IEEE Transactions On Power Electronics, Vol. 22, No. 3: 804-814

Hietpas, S., & Naden M. (2000). Automatic voltage regulator using an AC voltage-voltage converter, IEEE Transaction on Industry Application, Vol.36, No.1, pp. 33-38.

Hunyár, M. & Veszprémi, K. (2001). Pulse Width Modulated IGBT AC Chopper. Periodica Polytechnica Ser. El. Eng. Vol. 45, No. 3–4, pp. 159–178.

Kwon, B. & Jeong, G. (2002). Novel Line Conditioner with Voltage Up/Down Capability. IEEE Transactions On Industrial Electronics, Vol.49, No.5, pp.1110-1119.

Nan, J., Jun T., Yu, B., Xin G. & Liang, Y. (2010). Analysis and Control of Two Switches AC Chopper Voltage Regulator. Wseas Transactions on Circuits and Systems, Issue 4, Volume 9.

Saravanakumar, S., Mahendran, S., & Gnanambal I. (2015). Improving Voltage Regulation and Elimination of Harmonics in PWM AC Chopper Using Artificial Intelligent Technique. IJAICT Volume 1, Issue 8.

Sundareswaran, K., Rajasekar, N., & Sreedevi V. (2006). Performance comparison of capacitor-run induction motors supplied from AC voltage regulator and SPWM AC chopper. IEEE Transactions on Industrial Electronics, Vol.53, pp. 990-993.

Ursaru, O., Lucanu, M., Aghion, C. & Tigaeru, L. (2004). Three-Phase AC Chopper with IGBT's. 7th International Conference on Development and Application Systems Suceava, Romania, May 27–29.

Veszpremi, K. & Hunyar, M. (2000). New application fields of the PWM IGBT AC chopper. Eighth International Conference of Power Electronics and Variable Speed Drives, pp. 46-51.

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