Plastic buckling of thin flat rectangular isotropic plates under uniaxial in-plane loads

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Abstract. This study presents the analysis of plastic buckling of thin flat rectangular isotropic plates. To actualize this, the deformation theory of plasticity by Stowell's approach is used in expressing the governing differential equation, and this equation is modified by adopting the method of work principle based on the principle of conservation of energy. Taylor-Maclaurin series functions truncated at the fifth term is used in estimating the deflection functions. The analyzed plates are subjected to uniform uniaxial in-plane compression and the direction of the loading is in the longitudinal direction (x-axis). The three plate boundary conditions considered in this study are: four simply supported edges (SSSS); four clamped edges (CCCC); and two clamped edges along the x-axis and two simply supported edges along the y-axis (CSCS). The Taylor-Maclaurin series formulation satisfied each of the plate boundary conditions and resulted to a distinct deflection function for each plate. These deflection functions are substituted into the governing equation to obtain the critical plastic buckling loads. Values of the buckling coefficient, k, which is derived from the critical plastic buckling load equation, are calculated for aspect ratios, p, ranging from 0.1 to 1.0 in steps of 0.1, using values of moduli ratio, E_t/E_s, equal to 0.6, 0.7, 0.8, and 0.9. The results are compared with those of a previous investigation. The percentage differences of k with plastic buckling solutions for the different values of p and E_t/E_s of the plates ranged from −4.685% to 6.276%. It is shown that the technique proposed in this study is an alternative approximate method for analyzing the plastic buckling of thin rectangular isotropic plates under uniform uniaxial in-plane loads.

Keywords: deflection function, deformation plasticity theory, in-plane compression, plastic buckling, rectangular plate, Taylor-Maclaurin series.

1 INTRODUCTION

A plate is a plane structural member whose thickness is small in comparison with the other characteristic dimensions. Plates are used in engineering because of certain advantages such as their form efficiency, lighter structures produced from them, and economical advantages. Thin plate buckling involves bending in two planes and is generally more complicated than those of one–dimensional elements such as columns. Because of the two–dimensional buckling nature of thin plates, quantities such as deflections and bending moments are functions of two independent variables. Thin plate theories may be grouped according to their stress-strain relationships. Linear–elastic thin plate theories are based on Hooke's law which assumes that the relationship between stress and strain is linear, while non-linear plate theories consider more complex stress–strain relationships. When the applied load is increased beyond the elastic buckling load, the stresses on either or both axes exceed the elastic limit and the plate exhibits inelastic behaviour. Here, the linear-elastic thin plate theory is modified because Hooke's law is no longer valid. Various plastic plate theories have been proposed in literature to account for this inelastic effect.

The two major plasticity theories in thin plate buckling are the deformation theory of plasticity and the incremental (or flow) theory of plasticity. In the deformation theory, the strain that corresponds to a certain state of stress is entirely independent of the manner in which this stress state has been reached, while in the incremental theory, the strains and stresses are related by a function that depend on the loading path (Wang, 2006; Jones, 2009). Although the deformation theory contains fundamental mathematical inconsistencies which are not present in the incremental theory, results from the deformation theory tend to be in better agreement with experimental evidence (Chen, 2003; Aung, 2006). Generally, the deformation theory gives better prediction of buckling loads for rectangular plates (Wang, 2006).

The approaches which can be used to find solutions to the buckling problems of thin rectangular plates are the equilibrium or exact approach, the energy approach, and the numerical approach. It is difficult to obtain exact solutions for rectangular plates by the equilibrium approach except for a rectangular plate with four simply supported edges. This is because of the difficulty in assuming satisfactory deflection functions for rectangular plates of other boundary conditions (Ibearugbulem, 2012). Energy approaches such as the Rayleigh-Ritz method and Galerkin's method make use of approximate deflection functions. Many researchers applying energy approaches made use of the trigonometric series in formulating approximate deflection functions. Unfortunately, some boundary conditions make it difficult to use trigonometric series functions (Ugural, 1999; Ventsel and Krauthammer, 2001). In many solutions using the energy approaches, trial deflection functions are first selected. This means that the ability of the approximate solutions to converge to the exact solution depends on the closeness of the trial deflection function to the exact deflection function. The use of numerical methods in plate problem formulation is cumbersome and requires expertise in the use of computer. Some previous works on plastic buckling of plates include those by Iyengar (1988), Shen (1990), Chen (2003), Wang, et al. (2005), and Maarefdoust and Kadkhodayan (2013) using both the deformation and incremental theories. It is interesting to note that in literature, all the works on analysis of plastic buckling of plates by the equilibrium and energy approaches formulated deflection functions using trigonometric series. None of the solutions were obtained using polynomial series deflection functions. The limitation of trigonometric series functions in buckling analysis of rectangular plates is that it makes use of assumed shape functions, and it is difficult to assume satisfying shape functions for many plates of various boundary conditions (Ibearugbulem, et al., 2013). Polynomial deflection functions have the advantage of being applicable to rectangular plates with mixed boundary conditions, such as a plate where the opposite edges of the rectangular plate have different boundary conditions (Eziefula, 2014).

In this paper, the energy approach in the form of a work principle by the principle of conservation of energy is used. The governing equation for the plastic thin plate buckling is derived using the deformation theory of plasticity by Stowell's approach, and the approximate deflection functions are formulated using polynomial series in the form of Taylor-Maclurin series. The rectangular plates analyzed in this study are: four simply supported edges (SSSS); four clamped edges (CCCC); and, two clamped edges along the x-axis and two simply supported edges along the y-axis (CSCS). The thin plate is taken to be flat, homogenous, rectangular, and isotropic. The loading direction is along the x-axis as illustrated in Fig. 1. The SSSS, CCCC, and CSCS plates are shown in Figs. 2(a) – 2(c).

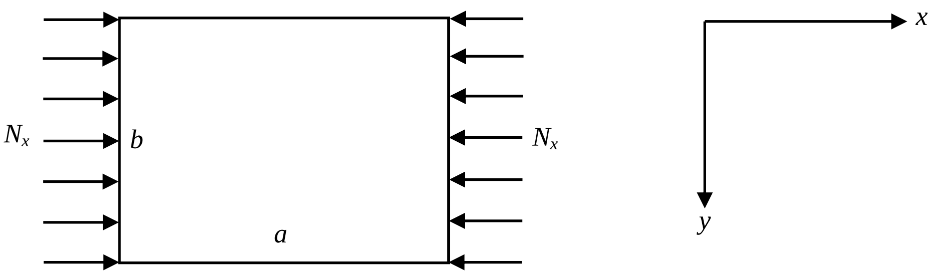

Fig. 1. Rectangular plate loaded on two opposite sides along the x-axis

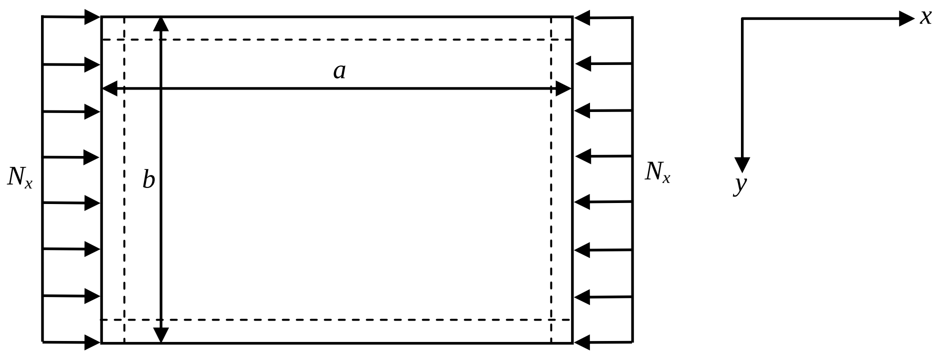

Fig. 2(a). SSSS rectangular plate under uniaxial compression along the *x*-axis

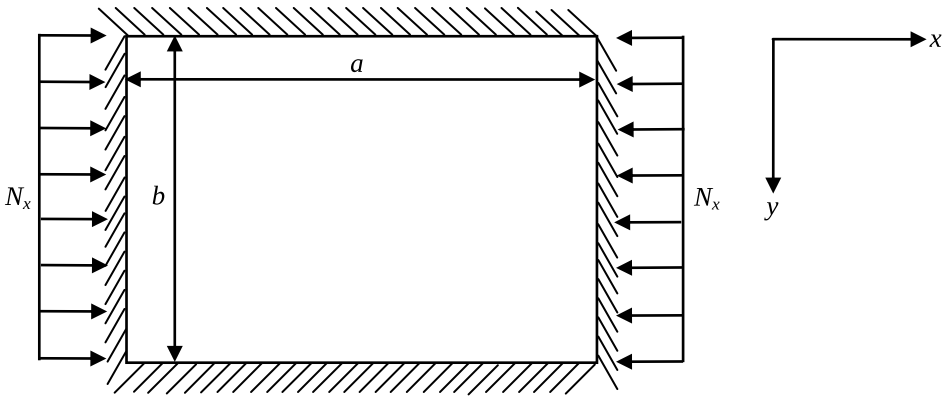

Fig. 2(b). CCCC rectangular plate under uniaxial compression along the *x*-axis

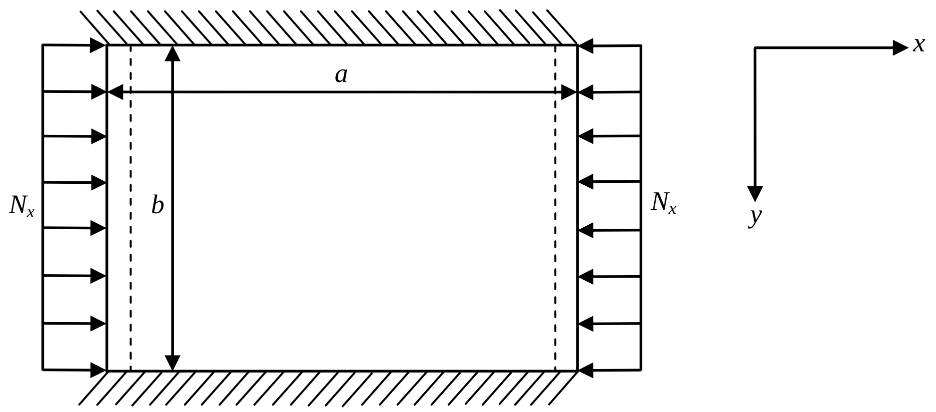

Fig. 2(c). CSCS rectangular plate under uniaxial compression along the *x*-axis

2 STOWELL'S APPROACH TO PLASTIC BUCKLING OF PLATES

Elbridge Z. Stowell, in 1948, proposed an approach for analyzing the plastic buckling of plates based on the deformation theory of plasticity. Stowell's approach is based on Shanley's approach to inelastic columns, where it is assumed that there is no strain reversal. Stowell's approach is a modification of Ilyushin's approach, which considers the plate to unload on one face as it buckles. In Stowell's approach however, it is assumed that no unloading takes place during plate buckling (Aung, 2006). Another modification is that while Ilyushin used the elastic flexural rigidity, D, in his mathematical formulation, Stowell used the plastic flexural rigidity, $\bar{D}$. The plastic flexural rigidity of the plate is expressed mathematically as shown in Eq. (1):

$$\bar{D} = \frac{E_s t^3}{9} \tag{1}$$

where E_s is the secant modulus and t is the thickness of the plate.

In comparison with the other approaches used in the deformation theory of plasticity, Stowell's approach gives lower buckling loads which are closer to results obtained from experiments for long rectangular plates made of materials such as aluminum alloys. Hence, Stowell's approach to the deformation theory of plasticity was used in this analysis.

In Stowell's approach, the following assumptions used in the elastic buckling of thin plates are adopted in the plastic buckling analysis (Iyengar, 1988):

(i) The effect of the vertical shear strains, γ_{xz} and γ_{yz}, and the normal strain, ε_z, are negligible.

(ii) The normal stress, σ_z, to the middle plane is neglected in the stress–strain relations.

(iii) The basic force and moment equilibrium equations are valid for the plastic buckling analysis.

(iv) The strain-displacement relations are also valid.

In deriving the inelastic constitutive equations, the following assumptions are made (Iyengar, 1988):

(i) The plate material is continuous and isotropic.

(ii) The principal axes of plastic stresses and strains coincide at all times.

(iii) The volume of the material remains constant (i.e. the material is incompressible).

(iv) The Poisson's ratio must increase from its elastic value to 0.5 for the plastic condition.

For a thin rectangular plate subjected to uniaxial compressive in-plane loads along the x-axis, Stowell (1948) derived the differential equation for the plastic buckling of the plate as:

$$\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{N_x}{\bar{D}}\frac{\partial^2 w}{\partial x^2} = 0 \tag{2}$$

where E_t is the tangent modulus, E_s is the secant modulus, w is the out-of-plane deflection, N_x is the in-plane compression along the x-axis, and $\bar{D}$ is the flexural rigidity of the plate in the inelastic range.

3 METHOD

3.1 Governing Equation

The theoretical development begins with the expression of the differential equation in non-dimensional coordinates. Expressing Eq. (2) in non-dimensional coordinates gives:

$$\frac{1}{p^2}\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial R^4} + 2\frac{\partial^4 w}{\partial R^2 \partial Q^2} + p^2\frac{\partial^4 w}{\partial Q^4} - \frac{N_x b^2}{\bar{D}}\frac{\partial^2 w}{\partial R^2} = 0 \tag{3}$$

where p is the aspect ratio, R is the non-dimensional coordinate along the x-axis, and Q is the non-dimensional coordinate along the x-axis. These parameters are defined as:

$$p = a/b \tag{4}$$

$$R = x/a\,, \qquad Q = y/b \tag{5}$$

where a and b are the length and width of the plate respectively.

Equation (3) is the Euler equilibrium of force equation for plastic buckling of a thin rectangular plate. Ibearugbulem, et al. (2013) used a technique for transforming Eq. (3) based on the principle of conservation of energy in a static continuum. This principle assumes that work done by the external applied loads is equal to the resistance of the plate. They multiplied the equation of equilibrium of force by the deflection and integrated the resulting equation in a closed domain. Applying this technique to Eq. (3), Eziefula (2014) derived the governing equation as:

$$N_x = \frac{\frac{\bar{D}}{b^2}\left\{\int_0^1\int_0^1\left[\frac{1}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)H\frac{\partial^4 H}{\partial R^4}+2H\frac{\partial^4 H}{\partial R^2\partial Q^2}+p^2 H\frac{\partial^4 H}{\partial Q^4}\right]\partial R\partial Q\right\}}{\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\,\partial R\partial Q} \quad (6)$$

where

$$w = AH \quad (7)$$

In Eqs. (6) and (7), H is the buckling curve expression, and A is the amplitude of the deflection function. Equation (6) is the governing equation for the plastic plate buckling.

3.2 Deflection Functions

The expression for the deflection function in non-dimensional parameters using Taylor-Maclaurin series formulation truncated at the fifth term was expressed by Ibearugbulem (2012) as:

$$w = A[(a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4)] \quad (8)$$

where a_0, a_1, a_2, a_3 and a_4 are the unknown constants of power series with respect to the *x*-direction and b_0, b_1, b_2, b_3 and b_4 are the unknown constants of power series with respect to the *y*-direction.

A rectangular plate has four edges and each of the edges could be simply supported (S), clamped (C) or free (F). The edge conditions of the four sides of the plate are represented using these symbols. To calculate the values of the constants a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3 and b_4 of a thin rectangular plate, the boundary conditions for all the edges of the plate are applied. For a simply supported edge, the values of the deflection and the bending moment at the boundaries are equal to zero. For a clamped edge, the values of the deflection and the slope at the boundaries are equal to zero. Applying the boundary conditions for each of the three plates analyzed in this paper, the deflection function of each plate is obtained by substituting the values of the constants of the power series into Eq. (8). From the deflection functions, the unique plate buckling expression of each plate is obtained as given from Eqs. (9) – (11).

SSSS plate:

$$H = (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) \quad (9)$$

CCCC plate:

$$H = (R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad (10)$$

CSCS plate:

$$H = (R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \quad (11)$$

3.3 Critical Plastic Buckling Loads

In calculating the critical plastic buckling load of the SSSS, CCCC, and CSCS plates, numerical values of the integrals in Eq. (6) for each plate are first determined. These values are then substituted into the plastic buckling equation i.e. Eq. (6) in order to obtain the critical plastic buckling load for each plate. From Eq. (6), these integrals are:

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial R^4}\partial R\partial Q$$

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q$$

$$\int_0^1\int_0^1 2H\frac{\partial^4 H}{\partial R^2\partial Q^2}\partial R\partial Q$$

$$\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q$$

The critical plastic buckling equations are derived for each plate by applying variational principles. These are given in Eqs. (12) - (14).

SSSS plate:

$$N_{x,CR}=\frac{\pi^2\overline{D}}{b^2}\left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.00027+1.00130p^2\right] \tag{12}$$

CCCC plate:

$$N_{x,CR}=\frac{\pi^2\overline{D}}{b^2}\left[\frac{4.25540}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.43172+4.25540p^2\right] \tag{13}$$

CSCS plate:

$$N_{x,CR}=\frac{\pi^2\overline{D}}{b^2}\left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+\ 2.43164+5.17330p^2\right] \tag{14}$$

where $N_{x,CR}$ is the critical plastic buckling load of the plate.

4 RESULTS AND DISCUSSION

The critical plastic buckling load, $N_{x,CR}$, is a function of the plate buckling coefficient, k. The relationship between $N_{x,CR}$ and k is:

$$N_{x,CR}\ =\frac{\pi^2\overline{D}}{b^2}k \tag{15}$$

The numerical values of the buckling coefficient are presented for each plate. The values of the buckling coefficient of each plate are calculated for aspect ratios, p, ranging from 0.1 to 1.0 in steps of 0.1, using values of moduli ratio, E_t/E_s, equal to 0.6, 0.7, 0.8, and 0.9.

SSSS plate:

For the SSSS plate, the result from the present study gives:

$$k=\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2.00027+1.00130p^2 \tag{16}$$

From Iyengar (1988), the result for the SSSS plate is:

$$k=\frac{1}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2+p^2 \tag{17}$$

CCCC plate:

For the CCCC plate, the result from the present study gives:

$$k=\frac{4.25540}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2.43172+4.25540p^2 \tag{18}$$

From Iyengar (1988), the result for the CCCC plate is:

$$k=\frac{4}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+2.66667+4p^2 \tag{19}$$

CSCS plate:
For the CSCS plate, the result from the present study gives:

$$k = \frac{1.00130}{p^2}\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right) + 2.43164 + 5.17330p^2 \qquad (20)$$

From Iyengar (1988), the result for the CSCS plate is:

$$k = \frac{1}{p^2}\left(\frac{1}{4} + \frac{3}{4}\frac{E_t}{E_s}\right) + 2.66667 + 5.33333p^2 \qquad (21)$$

The results obtained from Eqs. (16) - (21) for the SSSS, CCCC, and CSCS plates are presented in Tables 1(a)-(d), Tables 2(a)-(d) and Tables 3(a)-(d) respectively. The tables show the variation of the buckling coefficient (k) with the aspect ratio (p) and the moduli ratio (E_t/E_s), within the range of $0.1 \leq p \leq 1.0$ in steps of 0.1, and E_t/E_s equal to 0.6, 0.7, 0.8, and 0.9. For the variation k with p and E_t/E_s of the plates, it is generally observed that k reaches its maximum value when p is equal to 0.1 and that the value of k decreases as p approaches the numerical value of one. It is also observed that for a given value of p of a particular plate, the k has the maximum value for E_t/E_s equal to 0.9, and k reduces to a minimum value for E_t/E_s equal to 0.6. The results will coincide with the elastic buckling values when E_t/E_s is equal to unity.

Comparable solutions for the SSSS, CCCC, and CSCS plates are obtained from Iyengar (1988). Polynomial series deflection functions in the form of Taylor-Maclaurin series are used in this study while trigonometric series deflection functions are used in Iyengar (1988). For the SSSS and CCCC plates, it can be observed from Tables 1(a)-(d) and Tables 2(a)-(d) respectively that the solution obtained in this study and Iyengar's solution tend to converge as p increases from 0.1 to 1.0. For values of p of the SSSS plate, the percentage differences for the various values of E_t/E_s between the solution obtained in this work and Iyengar's solution are quite negligible. From the results of the SSSS plate, it is shown that the approximate deflection function by Taylor-Maclaurin series used in this work is closely approximate to those of the exact shape function. The solutions for the SSSS and CCCC plates obtained in this study are upper bound solutions. For the CSCS plate, it can be observed from Tables 3(a)-(d) that the solution obtained in this study and Iyengar's solution do not converge as p increases from 0.1 to 1.0. It can also be observed that for a given value of p for the CSCS plate, the percentage difference between the solutions presented in this study and Iyengar's solution increases as the value of E_t/E_s decreases.

5 CONCLUSIONS

Based on the results of this study, the following conclusions are drawn:

(i) The plastic buckling equation derived using a work principle based on the principle of conservation of energy was found to be satisfactory for expressing the governing equation of plasticity of a thin flat rectangular isotropic plate.

(ii) The Taylor-Maclaurin series formulation truncated at the fifth term satisfied each of the plate boundary conditions and resulted to a distinct deflection function for each rectangular plate.

(iii) The results of the study are in good agreement with solutions from a previous study.

(iv) The method proposed in this research work is recommended for use in plastic buckling analysis of thin flat rectangular isotropic plates. It is applicable in analysis of buckling of rectangular plates with mixed boundary conditions, such as a plate where the opposite edges of the rectangular plate have different boundary conditions.

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Table 1(a). Values of k for SSSS plate (E_t/E_s = 0.9).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	94.6305	94.5100	0.128
0.2	25.1954	25.1650	0.121
0.3	12.3815	12.3678	0.111
0.4	7.9490	7.9413	0.097
0.5	5.9554	5.9500	0.091
0.6	4.9335	4.9294	0.083
0.7	4.3811	4.3778	0.075
0.8	4.0883	4.0853	0.073
0.9	3.9547	3.9520	0.069
1.0	3.9277	3.9250	0.069

Table 1(b). Values of k for SSSS plate (E_t/E_s = 0.8).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	87.1208	87.0100	0.127
0.2	23.3179	23.2900	0.120
0.3	11.5471	11.5344	0.110
0.4	7.4800	7.4725	0.100
0.5	5.6550	5.6500	0.088
0.6	4.7249	4.7211	0.080
0.7	4.2279	4.2247	0.076
0.8	3.9710	3.9681	0.073
0.9	3.8621	3.8594	0.070
1.0	3.8527	3.8500	0.070

Table 1(c). Values of k for SSSS plate (E_t/E_s = 0.7).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	79.6110	79.5100	0.127
0.2	21.4405	21.4150	0.119
0.3	10.7127	10.7011	0.108
0.4	7.0105	7.0038	0.096
0.5	5.3546	5.3500	0.086
0.6	4.5163	4.5128	0.078
0.7	4.0746	4.0716	0.074
0.8	3.8536	3.8509	0.070
0.9	3.7694	3.7668	0.069
1.0	3.7776	3.7750	0.069

Table 1(d). Values of k for SSSS plate (E_t/E_s = 0.6).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	72.1012	72.0100	0.127
0.2	19.5631	19.5400	0.118
0.3	9.8783	9.8678	0.106
0.4	6.5412	6.5350	0.095
0.5	5.0542	5.0500	0.083
0.6	4.3077	4.3044	0.077
0.7	3.9213	3.9186	0.069
0.8	3.7363	3.7338	0.067
0.9	3.6766	3.6742	0.065
1.0	3.7025	3.7000	0.068

Table 2(a). Values of k for CCCC plate (E_t/E_s = 0.9).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	396.0988	372.7067	6.276
0.2	101.0081	95.3267	5.960
0.3	46.5508	44.1378	5.467
0.4	27.7141	26.4317	4.852
0.5	19.2406	18.4667	4.191
0.6	14.8977	14.3844	3.568
0.7	12.5500	12.1777	3.057
0.8	11.3056	11.0079	2.704
0.9	10.7382	10.4746	2.517
1.0	10.6234	10.3667	2.476

Table 2(b). Values of k for CCCC plate (E_t/E_s = 0.8).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	364.1833	342.7067	6.267
0.2	93.0292	87.8267	5.924
0.3	43.0046	40.8044	5.392
0.4	25.7194	24.5567	4.735
0.5	17.9639	17.2667	4.038
0.6	14.0111	13.5511	3.395
0.7	11.8987	11.5654	2.882
0.8	10.8069	10.5392	2.540
0.9	10.3441	10.1042	2.374
1.0	10.3042	10.0667	2.360

Table 2(c). Values of k for CCCC plate (E_t/E_s = 0.7).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	332.2678	312.7067	6.255
0.2	85.0503	80.3267	5.880
0.3	39.4584	37.4711	5.304
0.4	23.7247	22.6817	4.598
0.5	16.6873	16.0667	3.863
0.6	13.1246	12.7178	3.199
0.7	11.2473	10.9532	2.685
0.8	10.3082	10.0704	2.361
0.9	9.9501	9.7338	2.222
1.0	9.9851	9.7667	2.236

Table 2(d). Values of k for CCCC plate (E_t/E_s = 0.6).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	300.3522	282.7067	6.242
0.2	77.0714	72.8267	5.828
0.3	35.9123	34.1378	5.198
0.4	21.7300	20.8067	4.438
0.5	15.4107	14.8667	3.659
0.6	12.2381	11.8844	2.976
0.7	10.5960	10.3410	2.466
0.8	9.8095	9.6016	2.165
0.9	9.5561	9.3635	2.057
1.0	9.6659	9.4667	2.104

Table 3(a). Values of k for CSCS plate (E_t/E_s = 0.9).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	95.1036	95.2200	−0.122
0.2	25.7936	26.0050	−0.813
0.3	13.1884	13.4244	−1.758
0.4	9.0481	9.3013	−2.722
0.5	7.4298	7.7000	−3.509
0.6	6.8668	7.1561	−4.043
0.7	6.8568	7.1678	−4.339
0.8	7.1897	7.5253	−4.460
0.9	7.7655	8.1286	−4.467
1.0	8.5311	8.9250	−4.413

Table 3(b). Values of k for CSCS plate (E_t/E_s = 0.8).

p	k	Difference (%)

	Present Study	Iyengar (1988)	
0.1	87.5939	87.7200	−0.126
0.2	23.9162	24.1300	−0.886
0.3	12.3540	12.5911	−1.883
0.4	8.5788	8.8325	−2.872
0.5	7.1294	7.4000	−3.657
0.6	6.6582	6.9477	−4.167
0.7	6.7035	7.0147	−4.436
0.8	7.0724	7.4081	−4.532
0.9	7.6728	8.0361	−4.532
1.0	8.4560	8.8500	−4.452

Table 3(c). Values of k for CSCS plate ($E_t/E_s = 0.7$).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	80.0841	80.2200	−0.169
0.2	22.0388	22.2550	−0.971
0.3	11.5195	11.7578	−2.027
0.4	8.1094	8.3638	−3.042
0.5	6.8290	7.1000	−3.817
0.6	6.4496	6.7394	−4.300
0.7	6.5502	6.8616	−4.538
0.8	6.9551	7.2909	−4.606
0.9	7.5800	7.9435	−4.576
1.0	8.3809	8.7750	−4.491

Table 3(d). Values of k for CSCS plate ($E_t/E_s = 0.6$).

p	k		Difference (%)
	Present Study	Iyengar (1988)	
0.1	72.5744	72.7200	−0.200
0.2	20.1613	20.3800	−1.073
0.3	10.6851	10.9244	−2.191
0.4	7.6401	7.8950	−3.229
0.5	6.5286	6.8000	−3.991
0.6	6.2410	6.5311	−4.442
0.7	6.3970	6.7086	−4.645
0.8	6.8377	7.1738	−4.685
0.9	7.4873	7.8509	−4.631
1.0	8.3059	8.7000	−4.530