

Fuzzy Soft Matrix Topology and Fuzzy Soft Matrix Topology on $\vec{A}$

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Abstract. In this search we define the concept f_e - set by using the concept soft element , introduce an operations on f_e - sets to generate a fuzzy soft topology , study fuzzy soft matrix associated to f_e - set , study operations on fuzzy soft matrices to generate a new topology called fuzzy soft matrix topology and Fuzzy Soft Matrix Topology on $\vec{A}$.

Keywords: f_e – set , soft element, fuzzy soft topology , FSM_e - topology , Soft Matrix Topology on $\vec{A}$, FSM_e - open set, FSM_e - closed set, FSM_e - neighborhood .

1 INTRODUCTION

The first publication in fuzzy set theory by Zadeh 1965, Change In 1968 used fuzzy set theory to define a fuzzy topological space ,soft sets was introduced by the Russian Demetry Molodtsove 1999 as a general mathematical tool for dealing with uncertain objects, operations on soft set was introduced by P.K.maji , R.Biswas and A.R.Roy 2003, Muhammad Shabir and Munazza Naz 2011 introduce and study the concept of soft topological spaces over soft set and some related concepts such as soft subspace, Dariusz Wardowski 2013 introduce a new notion of soft element and establish its natural relation with soft sets in soft topological spaces , P.K.Maji, A.R. Roy , R. Biswas 2001 combined fuzzy sets and soft sets and introduced the concept of fuzzy soft sets, Tanay B. And Kandemir M.B. 2011 gave the topological structure of fuzzy soft sets , Young Yang and Chenli Ji define the concept fuzzy soft Matrices 2011 in this search we will define the concept f_e - set study fuzzy soft matrix associated to f_e - set , define a new operations on them to generate a new topologies : fuzzy soft topological space $(X, \vec{T}, E)$, fuzzy soft Matrix topological space $(X, \vec{T}_M, E)$ and fuzzy soft Matrix topological space $(X, \vec{T}_M, E)$ on $\vec{A}$.

2 Generating fuzzy soft topological space $(X, \vec{T}, E)$

Definition 2.1

The fuzzy soft element $\vec{x}$ is a soft element $\tilde{x} = (e, \{h\})$ associated to a number $\eta \in [0,1]$ represented as follow : $\vec{x} = (e, \{h^\eta\})$.

Definition 2.2

Let X be a universal set , E be a set of parameters , (F, E) be a soft set. If each soft element $\tilde{x}$ in (F, E) associated to a number $\eta \in [0,1]$ then resulting set is called a fuzzy soft set (simply f_e - set) denoted by (f_e, E) .

[i.e. $(f_e, E) = \{ \vec{x} : \vec{x} = (e_i, \{h_j^{\eta_j}\}) , \forall \tilde{x} = (e_i, \{h_j\}) \in (F, E) , \exists f_e : \vec{x} \rightarrow [0,1] \text{ s.t. } f_e(\vec{x}) = \eta_j \} \forall i, j \in \lambda , \eta_j \in [0,1] \}$.

Definition 2.3

The fact that $\tilde{x}$ is a fuzzy soft element of (f_e, E) will be denoted by $\tilde{x} \in (f_e, E)$.

Remark 2.4

Two fuzzy soft elements $\tilde{x} = (e_\alpha, \{h_\beta^\eta\})$, $\tilde{y} = (e_\gamma, \{h_\kappa^\nu\})$ are equal if $e_\alpha = e_\gamma$, $h_\beta = h_\kappa$, $\eta = \nu$.

Example 2.5

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$,

$(f_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.5}, h_2^{0.3}\})\}$, $\tilde{x} = (e_1, \{h_1^{0.5}\}) \in (f_e, E)$

and the fuzzy soft element $\tilde{y} = (e_1, \{h_2^{0.0}\}) \in (f_e, E)$, $\tilde{x} \neq \tilde{y}$

[but $(e_1, \{h_2\}) \notin (F, E)$] also $\tilde{z} = (e_1, \{h_2^{0.7}\}) \notin (f_e, E)$.

Remarks 2.6

1- A classical soft set (F, E) over X can be seen as an f_e -set (f_e, E) according to this manner for any $e \in E$, the image of e under f_e is defined as the characteristic function,

$$f_e(\tilde{x}) = \chi_{(F, E)}(\tilde{x}) = \begin{cases} 1, & \text{if } \tilde{x} \in (F, E); \\ 0, & \text{otherwise} \end{cases}.$$

2- f_e -sets are not necessary be a classical soft set.

Example 2.7

1- Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$

and let $(F, E) = \{(e_1, \{h_1\}), (e_2, \{h_1, h_2\})\}$ be a classical soft set

it can be written as f_e -set as follows

$(F, E) = (f_e, E) = \{(e_1, \{h_1^1, h_2^{0.0}\}), (e_2, \{h_1^1, h_2^1\})\}$

2- Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ then

$(g_e, E) = \{(e_1, \{h_1^{0.1}\}), (e_2, \{h_1^{0.7}\})\}$ is f_e -set but not classical soft set.

3- Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ then we can define an f_e -set on $\tilde{X}$ as follows

4- $(f_e, E) = \{(e_1, \{h_1^1, h_2^{0.0}\}), (e_2, \{h_1^1, h_2^1\})\}$.

Notification 2.8

$f_e(X)$ denoted to the set of all f_e -sets over a universe set X . $f_e(X)_E$ denoted to the set of all

f_e -sets over a universe set X in which all parameters E are the same which will be used in this section.

Definition 2.9

The f_e -complement of the f_e -set (f_e, E) denoted by

$(f_e, E)^c = \{\tilde{x}^c = (e_i, \{h_j^{1-\eta_j}\}) \mid \forall i, j \in \lambda, \eta_j \in [0, 1]\}$.

Definition 2.10

The f_e -set (f_e, E) generated by the soft set (F, E) is called the null f_e -set if each soft element $\tilde{x}$ in (F, E) associated to 0 (denoted by $\vec{0}$).

Definition 2.11

The f_e -set (f_e, E) generated by the soft set (F, E) is called the universal f_e -set if each soft element $\tilde{x}$ in (F, E) associated to 1 (denoted by $\vec{1}$).

Definition 2.12

Let f_e -sets (f_e, E) , (g_e, E) generated by the same soft set (F, E) s.t.

$(f_e, E) = \{\tilde{x}_f : \tilde{x}_f = (e_i, \{h_j^{\eta_j}\})\}$, $(g_e, E) = \{\tilde{x}_g : \tilde{x}_g = (e_i, \{h_j^{\delta_j}\})\}$

then (k_e, E) is said to be a f_e -subset of (g_e, E) if for each

$\tilde{x}_f = (e_i, \{h_j^{\eta_j}\}) \in (k_e, E)$ and $\tilde{x}_g = (e_i, \{h_j^{\delta_j}\}) \in (g_e, E)$, $\eta_j \leq \delta_j$

(denoted by $(k_e, E) \subseteq (g_e, E)$),

Definition 2.13

Let f_e -sets (f_e, E) , (g_e, E) generated by the same soft set (F, E) s.t.

$(f_e, E) = \{\tilde{x}_f : \tilde{x}_f = (e_i, \{h_j^{\eta_j}\})\}$, $(g_e, E) = \{\tilde{x}_g : \tilde{x}_g = (e_i, \{h_j^{\delta_j}\})\}$

then (k_e, E) is said to be f_e -equal of (g_e, E) if for each $\vec{x}_f = (e_i, \{h_j^{\eta_j}\}) \in (k_e, E)$ and $\vec{x}_g = (e_i, \{h_j^{\delta_j}\}) \in (g_e, E)$, $\eta_j = \delta_j$.

Definition 2.14

The f_e -union of two f_e -sets (f_e, E) , (g_e, E) generated by the same soft set (F, E) s.t.

$(f_e, E) = \{\vec{x}_f : \vec{x}_f = (e_i, \{h_j^{\eta_j}\})\}$, $(g_e, E) = \{\vec{x}_g : \vec{x}_g = (e_i, \{h_j^{\delta_j}\})\}$ is the f_e -set

$(k_e, E) = (f_e, E) \cup (g_e, E) = \{\vec{x}_k : \vec{x}_k = (e_i, \{h_j^{\nu_j}\})\}$, $\nu_j = \max\{\eta_j, \delta_j\}$.

Definition 2.15

The f_e -intersection of two f_e -sets (f_e, E) , (g_e, E) generated by the same soft set (F, E) s.t.

$(f_e, E) = \{\vec{x}_f : \vec{x}_f = (e_i, \{h_j^{\eta_j}\})\}$, $(g_e, E) = \{\vec{x}_g : \vec{x}_g = (e_i, \{h_j^{\delta_j}\})\}$ is the f_e -set

$(k_e, E) = (f_e, E) \cap (g_e, E) = \{\vec{x}_k : \vec{x}_k = (e_i, \{h_j^{\nu_j}\})\}$, $\nu_j = \min\{\eta_j, \delta_j\}$.

Example 2.16

Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$

and $(F, E) = \{(e_1, \{h_1\}), (e_2, \{h_1, h_2\})\}$, suppose two f_e -sets as follows :

$(f_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.3}, h_2^{0.3}\})\}$ and

$(g_e, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.0}\}), (e_2, \{h_1^{0.7}, h_2^{0.1}\})\}$ then

$(f_e, E) \cup (g_e, E) = (h_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.7}, h_2^{0.3}\})\}$

$(f_e, E) \cap (g_e, E) = (k_e, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.0}\}), (e_2, \{h_1^{0.3}, h_2^{0.1}\})\}$

Example 2.17

Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$,

let $(f_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.3}\})\}$

$(g_e, E) = \{(e_1, \{h_1^{0.9}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.7}\})\}$ be two f_e -sets generated by the same soft set $\tilde{X}$, since $(f_e, E) \subseteq (g_e, E)$ then

$(f_e, E) \cup (g_e, E) = \{(e_1, \{h_1^{0.9}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.7}\})\} = (g_e, E)$

$(f_e, E) \cap (g_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.3}\})\} = (f_e, E)$.

Remarks 2.18

(1) $(f_e, E) \cap (f_e, E)^c \neq \bar{\Phi}$.

(2) $(f_e, E) \cup (f_e, E)^c \neq \bar{X}$.

Example 2.19

Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$

$(f_e, E) = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.3}\})\}$ be f_e -set generated by the soft set $\tilde{X}$,

$(f_e, E)^c = \{(e_1, \{h_1^{0.5}, h_2^{1.0}\}), (e_2, \{h_1^{1.0}, h_2^{0.7}\})\}$ then

$(f_e, E) \cap (f_e, E)^c = \{(e_1, \{h_1^{0.5}, h_2^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.3}\})\} \neq \bar{\Phi}$

$(f_e, E) \cup (f_e, E)^c = \{(e_1, \{h_1^{0.5}, h_2^{1.0}\}), (e_2, \{h_1^{1.0}, h_2^{0.7}\})\} \neq \bar{X}$

Definition 2.20

The f_e -set (f_e, E) is said to be finite if the f_e -elements are finite and the

f_e -set (f_e, E) is said to be infinite if the f_e -elements are infinite.

Definition 2.21

More generally, for a family of f_e -sets, $\{(f_e, E)_\lambda : \lambda \in \Lambda$, where Λ is any index set},

the f_e -union is defined by:

$(h_e, E) = \bigcup_\lambda (f_e, E)_\lambda = \{\vec{x} : \vec{x} = (e_i, h_j^{\kappa_j})\}$, $\kappa_j = \sup_{\lambda \in \Lambda} \{\eta_j : \eta_j \text{ are the memberships of each } \vec{x} \text{ in the } f_e\text{-set } (f_e, E) \text{ and } j \in \xi\}$, where ξ is an infinite index set},

and the f_e -intersection is defined by:

$(g_e, E) = \bigcap_\lambda (f_e, E)_\lambda = \{\vec{x} : \vec{x} = (e_i, h_j^{\nu_j})\}$, where $\nu_j = \inf_{\lambda \in \Lambda} \{\eta_j : \eta_j \text{ are the memberships of each } \vec{x} \text{ in } f_e\text{-set } (f_e, E) \text{ and } j \in \xi\}$, where ξ is an infinite index set}.

Definition 2.22

Let X be non empty set , E be a set of parameters , let $\bar{T}$ be the collection of f_e - sets generated by the soft set $\bar{X}$, if $\bar{T}$ satisfies the following axioms :

- (1) $\bar{\Phi}, \bar{X}$ are in $\bar{T}$
- (2) The f_e -union of any members of f_e -sets in $\bar{T}$ belongs to $\bar{T}$.
- (3) The f_e -intersection of any two f_e -sets in $\bar{T}$ belong to $\bar{T}$.

Then $\bar{T}$ is called (fuzzy soft topology) (simply f_e - Topology) .

The triple $(X, \bar{T}, E)$ is called fuzzy soft topological space over X

(simply f_e - Topological space) , the f_e - sets of $\bar{T}$ are called f_e - open sets their complements are called f_e - closed sets .

Examples 2.23

(1) Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ then $\bar{T} = \{\bar{\Phi}, \bar{X}\}$ is the indiscrete f_e - Topology.

(2) Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$,

$(f_e, E) = \{(e_1, \{h_1^{0.7}, h_2^{0.9}, h_3^{0.9}\}), (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})\}$

then $\bar{T} = \{\bar{\Phi}, (f_e, E), \bar{X}\}$ be an f_e - Topology .

(3) Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2, e_3\}$,

$\bar{\Phi} = \{(e_1, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}\}), (e_3, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}\})\}$,

$\bar{X} = \{(e_1, \{h_1^1, h_2^1, h_3^1\}), (e_2, \{h_1^1, h_2^1, h_3^1\}), (e_3, \{h_1^1, h_2^1, h_3^1\})\}$

$(f_e, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.8}, h_3^{0.5}\}), (e_2, \{h_1^{0.8}, h_2^{0.1}, h_3^1\}), (e_3, \{h_1^{0.7}, h_2^{0.5}, h_3^{0.2}\})\}$

$(f_{1e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.4}, h_3^{0.1}\}), (e_2, \Phi), (e_3, \Phi)\}$

$(f_{2e}, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.5}, h_3^{0.5}\}), (e_2, \{h_1^{0.7}, h_2^{0.0}, h_3^{0.7}\}), (e_3, \{h_1^{0.6}, h_2^{0.1}, h_3^{0.1}\})\}$

$(f_{3e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.6}, h_3^{0.4}\}), (e_2, \{h_1^{0.1}, h_2^{0.1}, h_3^{0.9}\}), (e_3, \{h_1^{0.5}, h_2^{0.5}, h_3^{0.1}\})\}$

$(f_{4e}, E) = \{(e_1, \{h_1^{0.0}, h_2^{0.8}, h_3^{0.5}\}), (e_2, \{h_1^{0.8}, h_2^{0.0}, h_3^{0.1}\}), (e_3, \{h_1^{0.4}, h_2^{0.3}, h_3^{0.0}\})\}$

$(f_{5e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.8}, h_3^{0.5}\}), (e_2, \{h_1^{0.8}, h_2^{0.1}, h_3^{0.9}\}), (e_3, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.1}\})\}$

$(f_{6e}, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.8}, h_3^{0.5}\}), (e_2, \{h_1^{0.8}, h_2^{0.0}, h_3^{0.7}\}), (e_3, \{h_1^{0.6}, h_2^{0.3}, h_3^{0.1}\})\}$

$(f_{7e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.5}, h_3^{0.5}\}), (e_2, \{h_1^{0.7}, h_2^{0.0}, h_3^{0.7}\}), (e_3, \{h_1^{0.6}, h_2^{0.1}, h_3^{0.1}\})\}$

$(f_{8e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.8}, h_3^{0.5}\}), (e_2, \{h_1^{0.8}, h_2^{0.0}, h_3^{0.1}\}), (e_3, \{h_1^{0.4}, h_2^{0.3}, h_3^{0.0}\})\}$

$(f_{9e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.6}, h_3^{0.5}\}), (e_2, \{h_1^{0.7}, h_2^{0.1}, h_3^{0.9}\}), (e_3, \{h_1^{0.6}, h_2^{0.5}, h_3^{0.1}\})\}$

then $\bar{T} = \{\bar{\Phi}, \bar{X}, (f_e, E), (f_{1e}, E), (f_{2e}, E), (f_{3e}, E), (f_{4e}, E), (f_{5e}, E), (f_{6e}, E), (f_{7e}, E), (f_{8e}, E), (f_{9e}, E)\}$ be f_e - Topology.

Definition 2.24

Let X be non empty set , E be a set of parameters , let $\bar{T}$ be the collection of f_e - sets generated by the same soft set $(F, E) \neq \bar{X}$, if $\bar{T}$ satisfies the following axioms :

- (1) $\bar{\Phi}, (f_e, E)$ are in $\bar{T}$.
- (2) The f_e -union of any members of f_e -sets in $\bar{T}$ belongs to $\bar{T}$.
- (3) The f_e -intersection of any two f_e -sets in $\bar{T}$ belong to $\bar{T}$.

Then $\bar{T}$ is called (f_e - Topology on (f_e, E)). The triple $(X, \bar{T}, E)$ is called

(f_e - Topological space on (f_e, E)) , the f_e - sets of $\bar{T}$ are called f_e - open sets their complements are called f_e - closed sets .

Example 2.25

Let $X = \{h_1, h_2, h_3, h_4, h_5\}$, $E = \{e_1, e_2, e_3\}$,

$\bar{\Phi} = \{(e_1, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, h_4^{0.0}, h_5^{0.0}\}), (e_2, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, h_4^{0.0}, h_5^{0.0}\}), (e_3, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, h_4^{0.0}, h_5^{0.0}\})\}$,

is the f_e - null set .

$(f_e, E) = \{(e_1, \{h_1^{0.3}, h_2^{0.4}, h_3^{0.5}, h_4^{0.6}, h_5^{0.7}\}), (e_2, \{h_1^{0.8}, h_2^{0.5}, h_3^{0.9}, h_4^{0.5}, h_5^1\}), (e_3, \{h_1^{0.6}, h_2^{0.7}, h_3^{0.4}, h_4^{0.5}, h_5^{0.8}\})\}$,

$(f_{1e}, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.2}, h_3^{0.0}, h_4^{0.2}, h_5^{0.4}\}), (e_2, \{h_1^{0.5}, h_2^{0.1}, h_3^{0.4}, h_4^{0.1}, h_5^0\}), (e_3, \{h_1^{0.6}, h_2^{0.7}, h_3^{0.4}, h_4^{0.5}, h_5^{0.8}\})\}$,

$(f_{2e}, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.2}, h_3^{0.0}, h_4^{0.2}, h_5^{0.4}\}), (e_2, \{h_1^{0.5}, h_2^{0.1}, h_3^{0.4}, h_4^{0.1}, h_5^0\}), (e_3, \{h_1^{0.6}, h_2^{0.7}, h_3^{0.4}, h_4^{0.5}, h_5^{0.8}\})\}$,

$(e_3, \{h_1^{0.2}, h_2^{0.4}, h_3^{0.1}, h_4^{0.1}, h_5^{0.4}\})$
 $(f_{2e}, E) = \{(e_1, \{h_1^{0.3}, h_2^{0.4}, h_3^{0.2}, h_4^{0.5}, h_5^{0.6}\}), (e_2, \{h_1^{0.8}, h_2^{0.4}, h_3^{0.8}, h_4^{0.4}, h_5^{0.9}\}),$
 $(e_3, \{h_1^{0.5}, h_2^{0.7}, h_3^{0.2}, h_4^{0.3}, h_5^{0.8}\})\}$
 $(f_{3e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.3}, h_3^{0.1}, h_4^{0.3}, h_5^{0.5}\}), (e_2, \{h_1^{0.7}, h_2^{0.3}, h_3^{0.7}, h_4^{0.3}, h_5^{0.8}\}),$
 $(e_3, \{h_1^{0.4}, h_2^{0.6}, h_3^{0.1}, h_4^{0.2}, h_5^{0.6}\})\}$
 $(f_{4e}, E) = \{(e_1, \{h_1^{0.3}, h_2^{0.3}, h_3^{0.2}, h_4^{0.5}, h_5^{0.6}\}), (e_2, \{h_1^{0.8}, h_2^{0.3}, h_3^{0.7}, h_4^{0.4}, h_5^{0.8}\}),$
 $(e_3, \{h_1^{0.4}, h_2^{0.7}, h_3^{0.2}, h_4^{0.2}, h_5^{0.6}\})\}$
 $(f_{5e}, E) = \{(e_1, \{h_1^{0.2}, h_2^{0.4}, h_3^{0.1}, h_4^{0.3}, h_5^{0.5}\}), (e_2, \{h_1^{0.7}, h_2^{0.4}, h_3^{0.8}, h_4^{0.3}, h_5^{0.9}\}),$
 $(e_3, \{h_1^{0.5}, h_2^{0.6}, h_3^{0.1}, h_4^{0.3}, h_5^{0.8}\})\}$
 $\bar{T} = \{\bar{\Phi}, (f_e, E), (f_{1e}, E), (f_{2e}, E), (f_{3e}, E), (f_{4e}, E), (f_{5e}, E)\}$ be f_e -Topology on (f_e, E) , $(X, \bar{T}, E)$ is f_e -Topological space on (f_e, E) .

Remark 2.26

Let X be a universal set and E be a set of parameters then we can generate a

Definition 2.27

Let $(X, \bar{T}, E)$ be an f_e -Topological space over X ,
 $\bar{x} \in (g_e, E)$ then (g_e, E) is said to be f_e -neighborhood of $\bar{x}$ if there exist an
 f_e -open set (f_e, E) such that $\bar{x} \in (f_e, E) \subseteq (g_e, E)$.

Remark 2.28

Every f_e -open set is f_e -neighborhood but the converse is not necessary true.

Remark 2.29

Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$,
 $(f_e, E) = \{(e_1, \{h_1^{0.7}, h_2^{0.5}, h_3^{0.1}\}), (e_2, \{h_1^{0.5}, h_2^{0.3}, h_3^{0.0}\})\}$
 then $\bar{T} = \{\bar{\Phi}, (f_e, E), \bar{X}\}$ be an f_e -Topology over X , (f_e, E) is an f_e -open set it is also an
 f_e -neighborhood of $\bar{x} = (e_1, \{h_2^{0.5}\})$
 but the f_e -set $(g_e, E) = \{(e_1, \{h_1^{0.8}, h_2^{0.5}, h_3^{0.9}\}), (e_2, \{h_1^{0.5}, h_2^{0.7}, h_3^{1.0}\})\}$ is
 f_e -neighborhood of $\bar{x}$

3 Generating a fuzzy soft Matrix topological space $(X, \bar{T}_M, E)$

In this section we will study a fuzzy soft matrix associated to an f_e -set, the fuzzy soft matrices could be square or rectangular.

Definition 3.1

Let (f_e, E) be an infinite f_e -set over X , there exist the membership degree $f_{ej}((e_i, h_j)) = \eta_{ij}$, $\eta_{ij} \in [0, 1]$ then we can present all membership degrees by a table as follows:

	e_1	e_2	...	∞
h_1	$f_e(e_1, h_1) = \eta_{11}$	$f_e(e_2, h_1) = \eta_{12}$	...	∞
h_2	$f_e(e_1, h_2) = \eta_{21}$	$f_e(e_2, h_2) = \eta_{22}$	...	∞
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
∞	∞	∞	...	∞

The associated matrix represented by $\bar{A}_{\infty \times \infty} = \begin{bmatrix} \eta_{11} & \dots & \infty \\ \vdots & \ddots & \vdots \\ \infty & \dots & \infty \end{bmatrix}$

If (f_e, E) is finite f_e -set over X , then we can present all membership degrees by a table as follows:

	e_1	e_2	...	e_n
h_1	$f_e(e_1, h_1) = \eta_{11}$	$f_e(e_2, h_1) = \eta_{12}$	...	$f_e(e_n, h_1) = \eta_{1n}$
h_2	$f_e(e_1, h_2) = \eta_{21}$	$f_e(e_2, h_2) = \eta_{22}$	...	$f_e(e_n, h_2) = \eta_{2n}$

$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
h_m	$f_e(e_1, h_m) = \eta_{m1}$	$f_e(e_2, h_m) = \eta_{m2}$	$\dots$	$f_e(e_n, h_m) = \eta_{mn}$

and the associated matrix represented by $\vec{A}_{m \times n} = \begin{bmatrix} \eta_{11} & \dots & \eta_{1n} \\ \vdots & \ddots & \vdots \\ \eta_{m1} & \dots & \eta_{mn} \end{bmatrix}$

is called the fuzzy soft matrix of (f_e, E) over X (simply denoted by FSM_e).

Remark 3.2

In the general case (finite or infinite) $\vec{A}$ will be denoted to FSM_e and according to the previous definition each f_e -set (f_e, E) is uniquely characterized by a FSM_e $\vec{A}$ and vice versa.

Example 3.3

Let $X = \{h_1 = \text{wood}, h_2 = \text{plastic}, h_3 = \text{cartoon}, h_4 = \text{recycled materials}\}$ be a universal set of four materials. $E = \{e_1, e_2, e_3, e_4, e_5\}$ represent a set of parameters which is a five toys, each material contain a different ratio of these materials, the ratio of each material is between 0,1 (zero) mean the object does not contain that material. (one) says that the object is completely made of that material. Let $A = \{e_1, e_2, e_4\}$, $(f_e, A) = \{(e_1, \{h_1^{0.5}, h_2^{0.5}, h_3^{0.0}, h_4^{0.0}\}), (e_2, \{h_1^{0.3}, h_2^{0.2}, h_3^{0.2}, h_4^{0.3}\}), (e_3, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, h_4^{0.0}\}), (e_4, \{h_1^{0.5}, h_2^{0.1}, h_3^{0.3}, h_4^{0.1}\}), (e_5, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, h_4^{0.0}\})\}$ is a fuzzy soft set and the fuzzy soft matrix associated to this fuzzy soft set will be represented as follows:

$$\vec{A}_{4 \times 5} = \begin{bmatrix} 0.5 & 0.3 & 0.0 & 0.5 & 0.0 \\ 0.5 & 0.2 & 0.0 & 0.1 & 0.0 \\ 0.0 & 0.2 & 0.0 & 0.3 & 0.0 \\ 0.0 & 0.3 & 0.0 & 0.1 & 0.0 \end{bmatrix}.$$

Definition 3.5

The FSM_e whose elements are all 0 is called the *null- FSM_e* , denoted by $\vec{0}$.

Definition 3.6

The FSM_e whose elements are all 1 is called the *universal- FSM_e* , denoted by $\vec{U}$.

Definition 3.7

Let $\vec{A} = [a_{ij}] \in FSM_e(X)$ then complement of $\vec{A}$ is denoted by $\vec{A}^c = [c_{ij}]$

where $c_{ij} = 1 - a_{ij}$ for all i and j .

Definition 3.8

Let $\vec{A} \in FSM_e(X)_E$ then $\vec{x}_{ij} = f_e(e_i, h_j)$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$

are the FSM_e -elements of $\vec{A}$.

Definition 3.9

The fact that $\vec{x}$ is an FSM_e -element of $\vec{A}$ will be denoted by $\vec{x} \in \vec{A}$.

Definition 3.10

Let $\vec{A} = [a_{ij}]$, $\vec{B} = [b_{ij}] \in FSM_e(X)$ then $\vec{A}$ is a *fuzzy soft sub matrix* of $\vec{B}$, denoted by

$\vec{A} \subseteq \vec{B}$ if $a_{ij} \leq b_{ij} \forall i, j$.

Definition 3.11

Let $\vec{A} = [a_{ij}]$, $\vec{B} = [b_{ij}] \in FSM_e(X)$ then FSM_e -union of $\vec{A}$, $\vec{B}$ is defined by

$\vec{A} \cup \vec{B} = \vec{C} = [c_{ij}]$, $c_{ij} = \max\{a_{ij}, b_{ij}\} \forall i$ and j .

Definition 3.12

Let $\vec{A} = [a_{ij}]$, $\vec{B} = [b_{ij}] \in FSM_e(X)$ then FSM_e -intersection of $\vec{A}$ and $\vec{B}$ is defined by $\vec{A} \cap \vec{B} = \vec{C} = [c_{ij}]$ where $c_{ij} = \min\{a_{ij}, b_{ij}\} \forall i$ and j .

Example 3.13

Let $X = \{h_1, h_2, h_3\}$ be a universal set, $E = \{e_1, e_2, e_3\}$ be a set of parameters

$(f_e, E) = \{(e_1, \{h_1^{0.1}, h_2^{0.2}, h_3^{0.0}\}), (e_2, \{h_1^{0.2}, h_2^{0.2}, h_3^{0.2}\}), (e_3, \{h_1^{0.5}, h_2^{0.1}, h_3^{0.3}\})\}$ and

$(g_e, E) = \{ (e_1, \{ h_1^{0.2}, h_2^{0.3}, h_3^{0.0} \}), (e_2, \{ h_1^{0.2}, h_2^{0.5}, h_3^{0.2} \}), (e_3, \{ h_1^{0.5}, h_2^{0.1}, h_3^{0.5} \}) \}$

are two f_e - sets and the FSM_e associated to these

f_e - sets would be represented respectively as follows :

$$\vec{A}_{3 \times 3} = \begin{bmatrix} 0.1 & 0.2 & 0.5 \\ 0.2 & 0.2 & 0.1 \\ 0.0 & 0.2 & 0.3 \end{bmatrix}, \quad \vec{B}_{3 \times 3} = \begin{bmatrix} 0.2 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.1 \\ 0.0 & 0.2 & 0.5 \end{bmatrix}$$

$$\text{Then } \vec{A}_{3 \times 3} \cup \vec{B}_{3 \times 3} = \begin{bmatrix} 0.2 & 0.2 & 0.5 \\ 0.3 & 0.5 & 0.1 \\ 0.0 & 0.2 & 0.5 \end{bmatrix}$$

$$\vec{A}_{3 \times 3} \cap \vec{B}_{3 \times 3} = \begin{bmatrix} 0.1 & 0.2 & 0.5 \\ 0.2 & 0.2 & 0.1 \\ 0.0 & 0.2 & 0.3 \end{bmatrix}, \quad \vec{A}_{3 \times 3}^c = \begin{bmatrix} 0.9 & 0.8 & 0.5 \\ 0.8 & 0.8 & 0.9 \\ 1 & 0.8 & 0.7 \end{bmatrix}$$

Propositions 3.14

Let $\vec{A} = [a_{ij}]$, $\vec{B} = [b_{ij}]$, $\vec{C} = [c_{ij}]$, $\vec{0}$, $\vec{U}$ are all $FSM_e(X)_E$ then:

- 1- $\vec{0} \subseteq \vec{A}$, $\vec{A} \subseteq \vec{U}$, $\vec{A} \subseteq \vec{A}$.
- 2- $\vec{A} \subseteq \vec{B}$, $\vec{B} \subseteq \vec{C} \Rightarrow \vec{A} \subseteq \vec{C}$.
- 3- $\vec{A} \cap \vec{0} = \vec{0}$, $\vec{A} \cap \vec{U} = \vec{A}$.
- 4- $\vec{A} \cap \vec{B} = \vec{B} \cap \vec{A}$, $\vec{A} \cup \vec{B} = \vec{B} \cup \vec{A}$.
- 5- $\vec{A} \cup \vec{0} = \vec{A}$, $\vec{A} \cup \vec{U} = \vec{U}$.
- 6- $\vec{A} \cup \vec{A}^c = \vec{U}$, $\vec{A} \cap \vec{A}^c = \vec{0}$.
- 7- $\vec{0}^c = \vec{U}$, $\vec{A}^{c^c} = \vec{A}$.
- 8- $\vec{A} \cap \vec{A} = \vec{A}$.
- 9- $\vec{A} \cup \vec{A} = \vec{A}$.

Proposition 3.15

- 1- The infinite FSM_e - intersection of FSM_e is FSM_e .
- 2- The infinite FSM_e - union of FSM_e is FSM_e .

Proof Obviously.

Definition 3.16

Let X be non empty set, E be a set of parameters, let $\vec{T}_M$ be the collection of FSM_e - sets generated by the universal soft set $\vec{X}$ if $\vec{T}_M$ satisfies the following axioms :

- 1- $\vec{0}$, $\vec{U}$ belong to $\vec{T}_M$.
- 2- The FSM_e - union of any members of FSM_e - sets in $\vec{T}_M$ belongs to $\vec{T}_M$.
- 3- The FSM_e - intersection of any two FSM_e - sets in $\vec{T}_M$ belong to $\vec{T}_M$.

Then $\vec{T}_M$ is said to be *fuzzy soft matrix topology* (denoted FSM_e -Topology). The triple $(X, \vec{T}_M, E)$ is called a *FSM_e topological space* over X (denoted by FSM_e -T.S.), the members of $\vec{T}_M$ are called *FSM_e - open sets* their complements are called *FSM_e -closed sets*.

Definition 3.17

Let X be non empty set , E be a set of parameters , let $\vec{T}_M$ be the collection of FSM_e - sets , $\vec{A} \subset \vec{U}$, if $\vec{T}_M$ satisfies the following axioms :

1- $\vec{0}, \vec{A}$ belong to $\vec{T}_M$.

2- The FSM_e - union of any members of FSM_e - sets in $\vec{T}_M$ belongs to $\vec{T}_M$.

3- The FSM_e - intersection of any two FSM_e - sets in $\vec{T}_M$ belong to $\vec{T}_M$.

Then $\vec{T}_M$ is (FSM_e - Topology on $\vec{A}$). The triple $(X, \vec{T}_M, E)$ is called a FSM_e topological space on $\vec{A}$ (denoted by FSM_e - T.S. on $\vec{A}$) , the members of $\vec{T}_M$ are called FSM_e - open sets their complements are called FSM_e - closed sets .

Examples 3.18

1-Let $X = \{h_1, h_2, h_3, \dots\}$, $E = \{e_1, e_2, e_3, \dots\}$,

$\vec{\Phi} = \{ (e_1, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, \dots\}), (e_2, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, \dots\}), (e_3, \{h_1^{0.0}, h_2^{0.0}, h_3^{0.0}, \dots\}), \dots \}$ and

$\vec{X} = \{ (e_1, \{h_1^1, h_2^1, h_3^1, \dots\}), (e_2, \{h_1^1, h_2^1, h_3^1, \dots\}), (e_3, \{h_1^1, h_2^1, h_3^1, \dots\}), \dots \}$

are two f_e - set generated by the universal soft set $\vec{X}$, $\vec{T}_i = \{\vec{\Phi}, \vec{X}\}$ is the indiscrete f_e -Topology. The FSM_e - sets associated to f_e - set will be represented respectively as follows :

$$\vec{0} = \begin{bmatrix} 0 & \dots & \infty \\ \vdots & \ddots & \vdots \\ \infty & \dots & \infty \end{bmatrix}, \vec{U} = \begin{bmatrix} 1 & \dots & \infty \\ \vdots & \ddots & \vdots \\ \infty & \dots & \infty \end{bmatrix}$$

$\vec{T}_{Mi} = \{\vec{0}, \vec{U}\}$ is the indiscrete FSM_e - Topology .

2- Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2, e_3\}$, let $(f_e, E) = \{ (e_1, \{h_1^{0.1}, h_2^{0.2}, h_3^{0.0}\}), (e_2, \{h_1^{0.2}, h_2^{0.2}, h_3^{0.2}\}), (e_3, \{h_1^{0.5}, h_2^{0.1}, h_3^{0.3}\}) \}$ represented as follows

$$\vec{A}_{3 \times 3} = \begin{bmatrix} 0.1 & 0.2 & 0.5 \\ 0.2 & 0.5 & 0.1 \\ 0.0 & 0.2 & 0.3 \end{bmatrix} \text{ then } \vec{T}_M = \{\vec{0}_{3 \times 3}, \vec{U}_{3 \times 3}, \vec{A}_{3 \times 3}\} \text{ be a } FSM_e\text{-Topology} .$$

3- In this example we will generate a $\vec{T}_M$ - Topology . Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2, e_3\}$ then from example

2.23.(3) all f_e - sets can be represented respectively as an 3×3 matrices as follows :

$$\vec{0} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, \vec{U} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \vec{A} = \begin{bmatrix} 0.2 & 0.8 & 0.7 \\ 0.8 & 0.1 & 0.5 \\ 0.5 & 0.1 & 0.2 \end{bmatrix},$$

$$\vec{A}_1 = \begin{bmatrix} 0.2 & 0.0 & 0.0 \\ 0.4 & 0.0 & 0.0 \\ 0.1 & 0.0 & 0.0 \end{bmatrix}, \vec{A}_2 = \begin{bmatrix} 0.1 & 0.7 & 0.0 \\ 0.5 & 0.0 & 0.1 \\ 0.5 & 0.7 & 0.1 \end{bmatrix}, \vec{A}_3 = \begin{bmatrix} 0.2 & 0.1 & 0.5 \\ 0.6 & 0.1 & 0.5 \\ 0.4 & 0.9 & 0.1 \end{bmatrix},$$

$$\vec{A}_4 = \begin{bmatrix} 0.0 & 0.8 & 0.4 \\ 0.8 & 0.0 & 0.3 \\ 0.5 & 0.1 & 0.0 \end{bmatrix}, \vec{A}_5 = \begin{bmatrix} 0.2 & 0.8 & 0.5 \\ 0.8 & 0.1 & 0.3 \\ 0.5 & 0.9 & 0.1 \end{bmatrix}, \vec{A}_6 = \begin{bmatrix} 0.1 & 0.8 & 0.6 \\ 0.8 & 0.0 & 0.3 \\ 0.5 & 0.7 & 0.1 \end{bmatrix}$$

$$\vec{A}_7 = \begin{bmatrix} 0.2 & 0.7 & 0.6 \\ 0.5 & 0.0 & 0.1 \\ 0.5 & 0.7 & 0.1 \end{bmatrix}, \vec{A}_8 = \begin{bmatrix} 0.2 & 0.8 & 4.0 \\ 0.8 & 0.0 & 3.0 \\ 0.5 & 0.1 & 0.0 \end{bmatrix}, \vec{A}_9 = \begin{bmatrix} 0.2 & 0.7 & 0.6 \\ 0.6 & 0.1 & 0.5 \\ 0.5 & 0.9 & 0.1 \end{bmatrix}$$

$\vec{T}_M = \{\vec{0}, \vec{U}, \vec{A}, \vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{A}_4, \vec{A}_5, \vec{A}_6, \vec{A}_7, \vec{A}_8, \vec{A}_9\}$ is FSM_e - Topology $(X, \vec{T}_M, E)$ be FSM_e - Topological space.

4- In this example we will generate a $\vec{T}_M$ - Topology on $FSM_e \vec{A}_{5 \times 3}$, from example (2.25) all f_e - sets can be represented respectively as an 5×3 matrices as follows :

$$\vec{0} = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}, \vec{A} = \begin{bmatrix} 0.3 & 0.8 & 0.6 \\ 0.4 & 0.5 & 0.7 \\ 0.5 & 0.9 & 0.4 \\ 0.6 & 0.5 & 0.5 \\ 0.7 & 1 & 0.8 \end{bmatrix}, \vec{A}_1 = \begin{bmatrix} 0.1 & 0.5 & 0.2 \\ 0.2 & 0.1 & 0.4 \\ 0.0 & 0.4 & 0.1 \\ 0.2 & 0.1 & 0.1 \\ 0.4 & 0.0 & 0.4 \end{bmatrix}$$

$$\vec{A}_2 = \begin{bmatrix} 0.3 & 0.8 & 0.5 \\ 0.4 & 0.4 & 0.7 \\ 0.2 & 0.8 & 0.2 \\ 0.5 & 0.4 & 0.3 \\ 0.6 & 0.9 & 0.8 \end{bmatrix}, \vec{A}_3 = \begin{bmatrix} 0.2 & 0.7 & 0.4 \\ 0.3 & 0.3 & 0.6 \\ 0.1 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.2 \\ 0.5 & 0.8 & 0.6 \end{bmatrix}, \vec{A}_4 = \begin{bmatrix} 0.3 & 0.8 & 0.4 \\ 0.3 & 0.3 & 0.7 \\ 0.2 & 0.7 & 0.2 \\ 0.5 & 0.4 & 0.2 \\ 0.6 & 0.8 & 0.6 \end{bmatrix}$$

$$\vec{A}_5 = \begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.4 & 0.4 & 0.6 \\ 0.1 & 0.8 & 0.1 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.9 & 0.8 \end{bmatrix}.$$

Then $\vec{T}_M = \{\vec{0}, \vec{A}, \vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{A}_4, \vec{A}_5\}$ is FSM_e - Topology on $\vec{A}_{5 \times 3}$, $(X, \vec{T}_M, E)$ be FSM_e - Topological space on $\vec{A}_{5 \times 3}$, each FSM_e - set in $\vec{T}_M$ is FSM_e - open set.

Definition 3.19

Let $(X, \vec{T}_M, E)$ be a FSM_e - Topological space over X , $\vec{G} \in FSM_e(X)$, $\vec{x} \in \vec{G}$ then $\vec{G}$ is said to be a FSM_e - neighborhood of $\vec{x}$ if there exist a FSM_e - open set $\vec{A}$ such that $\vec{x} \in \vec{A} \subseteq \vec{G}$.

Remark 3.20

Every FSM_e - open is FSM_e - neighborhood but the converse is not necessary true.

Example 3.21

From example (3.18.(4)) the 5×3 FSM_e $\vec{A}_5 = \begin{bmatrix} 0.2 & 0.7 & 0.5 \\ 0.4 & 0.4 & 0.6 \\ 0.1 & 0.8 & 0.1 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.9 & 0.8 \end{bmatrix}$

is FSM_e - open it is FSM_e - neighborhood of each of its element, but the 5×3 FSM_e

$$\vec{D} = \begin{bmatrix} 0.3 & 0.7 & 0.5 \\ 0.4 & 0.4 & 0.6 \\ 0.1 & 0.8 & 0.1 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.9 & 0.8 \end{bmatrix}$$

is FSM_e - neighborhood of each of its element but not FSM_e - open

Since $\vec{D} \notin \vec{T}_M$.

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