Improved Symbol to Symbol Detection of Inter-Symbol Interference in Channel Equalization

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Abstract: Inter-symbol interference (ISI) is a common practical impairment found in many transmission and storage systems, including voice-band modems, digital subscriber loop data transmission, storage disks, digital mobile radio channels, digital microwave channels, and fiber-optic cables. In a receiver for detection of a succession of messages, output sample is the input to the same one-shot detector that would be used on an AWGN (Adaptive White Gaussian Noiseless) channel without ISI. Interference between successive transmissions, or inter-symbol interference, can degrade the performance of symbol-by-symbol detection. This performance degradation increases as the symbol rate increases in most communication channels. Communication engineers use equalization methods to mitigate the effects of the inter-symbol interference. There are several inter-symbol interference and equalization methods, which amount to different structures for the receiver. An alternative (suboptimal) receiver can detect each of the successive K messages independently as presented in this paper. The symbol-by-symbol (SBS) detector, while optimum for the AWGN channel, will not be a maximum-likelihood estimator for the sequence of messages. The bank of matched filters, found by Gram-Schmitt decomposition of the set of channel output waveforms, precedes a block detector that determines the K-dimensional vector symbol transmitted. The complexity would become large or infinite as K becomes large or infinite for the block detector.

Keywords: Inter-Symbol Interference, Symbol-by-Symbol, Detection, Equalization, Channel, Receiver.

1. INTRODUCTION

Inter-symbol interference (ISI) is a common practical impairment found in many transmission and storage systems, including voice-band modems, digital subscriber loop data transmission, storage disks, digital mobile radio channels, digital microwave channels, and even fiber-optic cables. Practical transmission systems send sequence of messages, one after another for one-shot analysis to apply. These successive transmissions must not interfere with one another. In practice, successive transmissions do often interfere with one another, especially as they are spaced more closely together to increase the data transmission rate. The interference between successive transmissions is called inter-symbol interference. ISI can severely complicate the implementation of an optimum detector.

In a receiver for detection of a succession of messages as shown in figure 1, the matched filter outputs are processed by the receiver, which outputs samples, z_k, that estimate the symbol transmitted at time k, $\hat{x}_k$. Each receiver output sample is the input to the same one-shot detector that would be used on an AWGN (Adaptive White Gaussian Noiseless) channel without ISI. This symbol-by-symbol (SBS) detector, while optimum for the AWGN channel, will not be a maximum-likelihood estimator for the sequence of messages. However, for a properly designed receiver, the combination of receiver and detector may work nearly as well as an optimum detector with far less complexity. The objective of the receiver is to improve the performance of this simple SBS detector.

Communication engineers use equalization methods to mitigate the effects of the inter-symbol interference. An equalizer is essentially the content of the receiver box of figure 1. There are several inter-symbol interference and equalization methods, which amount to different structures for the receiver box. These equalization methods try to convert a band-limited channel with ISI into one that appears memoryless, hopefully synthesizing a new AWGN-like channel at the receiver output. The designer can then analyze the resulting memoryless, equalized channel (Wong et al).

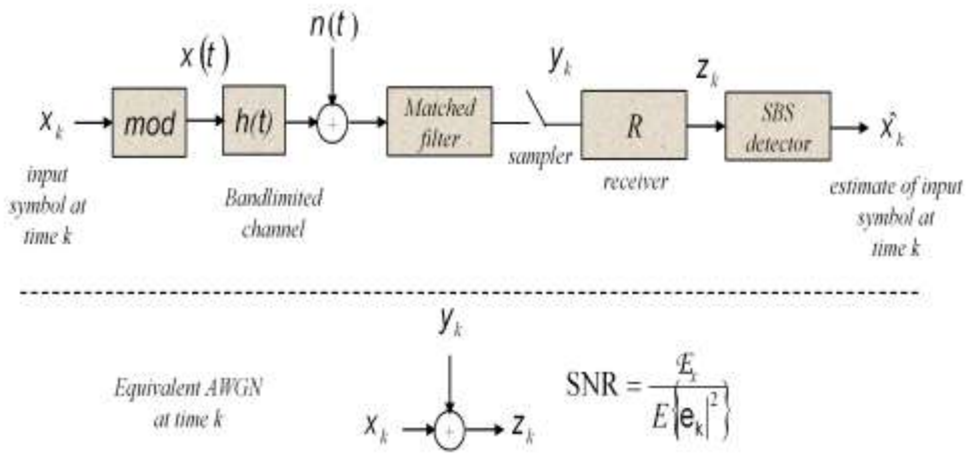

Figure 1: The band-limited channel with receiver and SBS detector.

From figure 1, the objective of the receiver will be to convert the channel into an equivalent AWGN at each time *k*, independent of all other times *k*. An AWGN detector may then be applied to the derived channel, and performance computed readily using the gap approximation or other known formulae with the SNR of the derived AWGN channel. There may be loss of optimality in creating such an equivalent AWGN, which will be measured by the SNR of the equivalent AWGN with respect to the best value that might be expected otherwise for an optimum detector.

2. INTER-SYMBOL INTERFERENCE AND RECEIVERS FOR SUCCESSIVE MESSAGE TRANSMISSION

Most communication systems re-use the channel to transmit several messages in succession. The message transmissions are separated by *T* units in time, where *T* is called the symbol period, and 1/*T* is the symbol rate. The data rate for a communication system that sends one of *M* possible messages every *T* time units is:

$$R = \frac{\log_2(M)}{T} = \frac{b}{T} \tag{1}$$

To increase the data rate in a design, either *b* can be increased (which requires more signal energy) or *T* can be decreased. Decreasing *T* narrows the time between message transmissions and thus increases inter-symbol interference on any band-limited channel (Kaur, R. et al (2013)).

The transmitted signal $x(t)$ corresponding to *K* successive transmissions is:

$$x(t) = \sum_{k=0}^{K-1} x_k(t - kT) \tag{2}$$

The *K* successive transmissions could be considered an aggregate or "block" symbol, $x(t)$, conveying one of M^K possible messages. The receiver could attempt to implement MAP or ML detection for this new transmission system with M^K messages. A Gram-Schmidt decomposition on the set of M^K signals would then be performed and an optimum detector designed accordingly. Such an approach has complexity that grows exponentially (in proportion to M^K) with the block message length *K*. That is, the optimal detector might need M^K matched filters, one for each possible transmitted block symbol. As $K \rightarrow 1$, the complexity can become too large for practical implementation. It is possible to compute the `a posteriori probability function with less than exponentially growing complexity.

An alternative (suboptimal) receiver can detect each of the successive K messages independently. Such detection is called a symbol-by-symbol (SBS) detection as shown in figure 2. The bank of matched filters, found by Gram-Schmitt decomposition of the set of (noiseless) channel output waveforms (of which it can be shown that K dimensions are sufficient only if $N = 1$, complex or real), precedes a block detector that determines the K-dimensional vector symbol transmitted. The complexity would become large or infinite as K becomes large or infinite for the block detector. The lower system in figure 2 has a single matched filter to the channel, with output sampled K times, followed by a receiver and an SBS detector. The later system has fixed (and lower) complexity per symbol/sample, but may not be optimum. Interference between successive transmissions, or inter-symbol interference (ISI), can degrade the performance of symbol-by-symbol detection (Singal, T. L. (2011)).

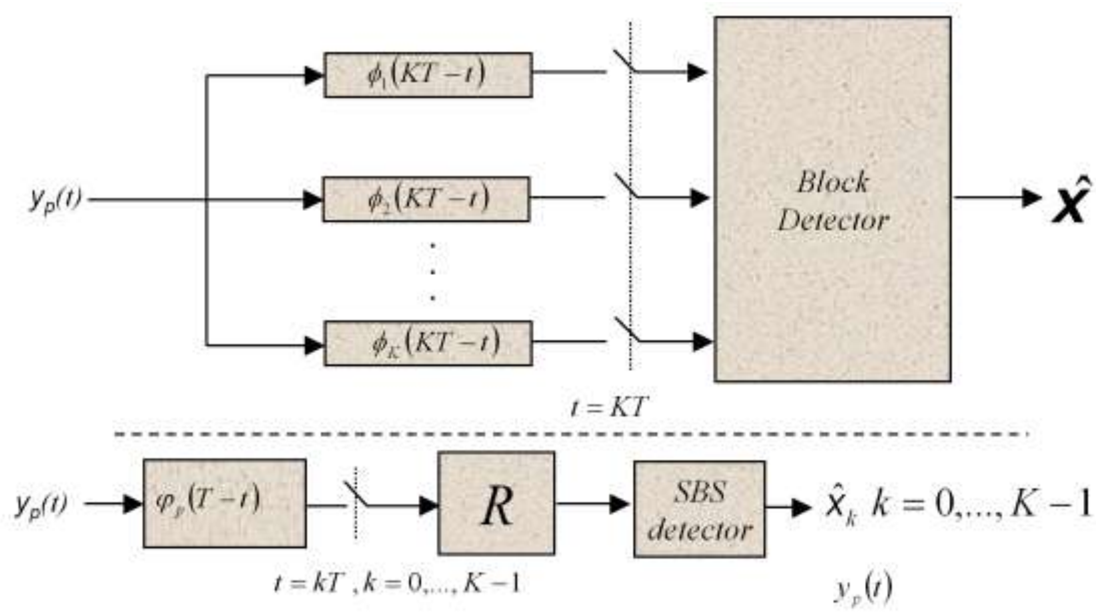

Figure 2: Comparison of Block and SBS detectors for successive transmission of K messages

. This performance degradation increases as T decreases (or the symbol rate increases) in most communication channels. The designer mathematically analyzes ISI by rewriting eqn. 2 as:

$$x(t) = \sum_{k=0}^{K-1} \sum_{n=1}^{N} x_{kn} \varphi(t - kT) \qquad (3)$$

where the transmissions $x_k(t)$ are decomposed using a common orthonormal basis set $\{\varphi_n(t)\}$. In (3), where $\varphi_n(t - kT)$ and $\varphi_m(t - lT)$ may be non-orthogonal when $k \neq l$ and in some cases translates of the basic functions are orthogonal. For instance, in Quadrature Amplitude Modulation (QAM), the two band-limited basis functions:

$$\varphi_1(t) = \sqrt{\frac{2}{T}} \cos\left(\frac{m\pi t}{T}\right) . \sin c\left(\frac{t}{T}\right) \qquad (4)$$

$$\varphi_2(t) = -\sqrt{\frac{2}{T}} \sin\left(\frac{m\pi t}{T}\right) . \sin c\left(\frac{t}{T}\right) \qquad (5)$$

and the baseband equivalent

$$\varphi(t) = \frac{1}{\sqrt{T}} \sin c\left(\frac{t}{T}\right) \qquad (6)$$

(with *m* a positive integer) are orthogonal for all integer-multiple-of-*T* time translations. Here, the successive transmissions, when sampled at time instants *kT*, are free of ISI, and transmission is equivalent to a succession of "one-shot" uses of the channel. In this case symbol-by-symbol detection is optimal, and the MAP detector for the entire block of messages is the same as a MAP detector used separately for each of the *K* independent transmissions. Signal sets for data transmission are usually designed to be orthogonal for any translation by an integer multiple of symbol periods. Most linear AWGN channels, however, are more accurately modeled by a filtered AWGN channel. The filtering of the channel alters the basis functions so that at the channel output the filtered basis functions are no longer orthogonal. The channel thus introduces ISI (Orlic, V. D et al (2009)).

3. THE INTER-SYMBOL CHANNEL MODEL

Figure 3 shows a model for linear Inter-Symbol Interference (ISI) channels. For this model, x_k is scaled by $\|p\|$ to form $x_{p,k}$ so that $\mathcal{E}x_p = \mathcal{E}x.\ \|p\|^2$. The additive noise is white Gaussian, although correlated Gaussian noise can be included by transforming the correlated-Gaussian-noise channel into an equivalent white Gaussian noise channel. The channel output $y_p(t)$ is passed through a matched filter $\varphi_p * (-t)$ to generate $y(t)$. $y(t)$ is then sampled at the symbol rate and processed by a discrete time receiver.

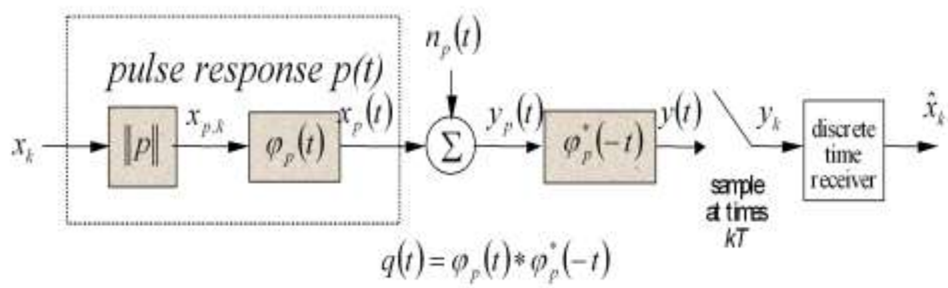

Figure 3: The Inter-Symbol Interference Channel model

The discrete-time signal samples $y_k = y(kT)$ as in figure 3 are sufficient to represent the continuous-time ISI-model channel output $y(t)$, if $0 < \|p\| < \infty$, that is, a receiver with minimum power can be designed that uses only the samples y_k.

Now define; $$\varphi_{p,k}(t) \cong \varphi_p(t - kT) \qquad (7)$$

where $\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ is a linearly independent set of functions. The set $\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ is related to a set of orthogonal basis functions $\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ by an invertible transformation Γ (use Gram-Schmidt an infinite number of times). The transformation and its inverse are:

$$\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)} = \Gamma\left(\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty}\right) \qquad (8)$$

$$\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)} = \Gamma^{-1}\left(\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty}\right) \qquad (9)$$

where Γ is the invertible transformation. In Figure 4, the transformation outputs are the filter samples $y(kT)$. The infinite set of filters $\{\varphi *_{p,k}(-t)\}_{k\in(-\infty,\infty)}$ followed by Γ^{-1} is equivalent to an infinite set of matched filters to $\{\varphi *_{p,k}(-t)\}_{k\in(-\infty,\infty)}$. By (7), this last set is equivalent to a single matched filter $\varphi *_{p}(-t)$, whose output is sampled at $t = kT$ to produce $y(kT)$. Since the set $\{\varphi_{p,k}(t)\}_{k\in(-\infty,\infty)}$ is orthonormal, the set of sampled filter outputs in Figure 4 are sufficient to represent $y_p(t)$. Since Γ^{-1} is invertible (inverse is Γ), then the sampled matched filter output $y(kT)$ is a sufficient representation of the ISI-channel output $y_p(t)$.

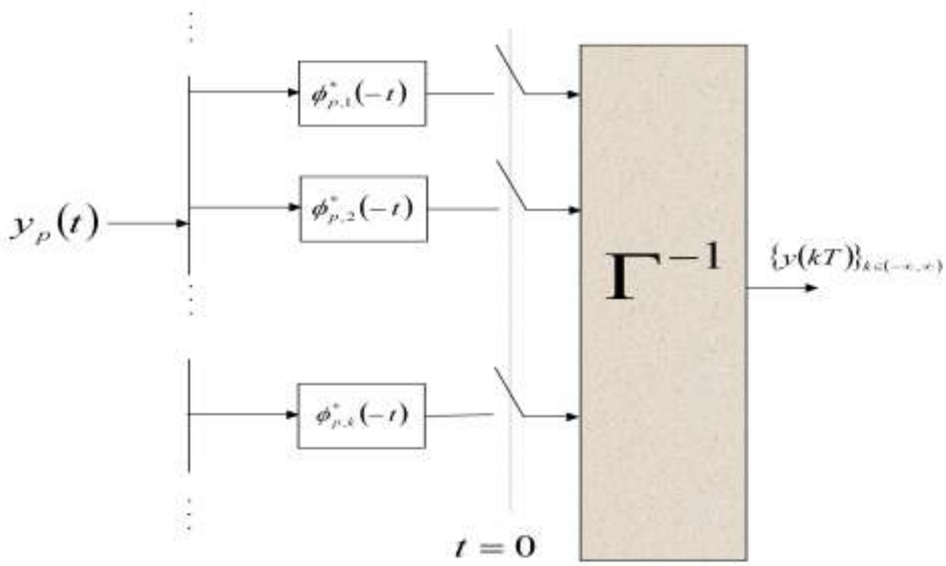

Figure 4: Equivalent diagram of ISI-channel model matched-filter/sampler

Referring to figure 3,

$$y(t) = \sum_k \|p\|.x_k q(t-kT) + n_p(t) * \varphi^*{}_p(-t) \qquad (10)$$

where

$$q(t) \equiv \varphi_p(t) * \varphi^*{}_p(-t) = \frac{p(t) * p^*(-t)}{\|p\|^2} \qquad (11)$$

The deterministic autocorrelation function q(t) is ($q^*(-t) = q(t)$). Also, $q(0) = 1$, so the symbol x_k passes at time kT to the output with amplitude scaling $\|p\|$. The function q(t) which can exhibit Inter-symbol Interference is as illustrated in figure 5. The plotted $q(t)$ corresponds to q_k = [−0.1159, 0 .2029, 1, 0 .2029, −0.1159].

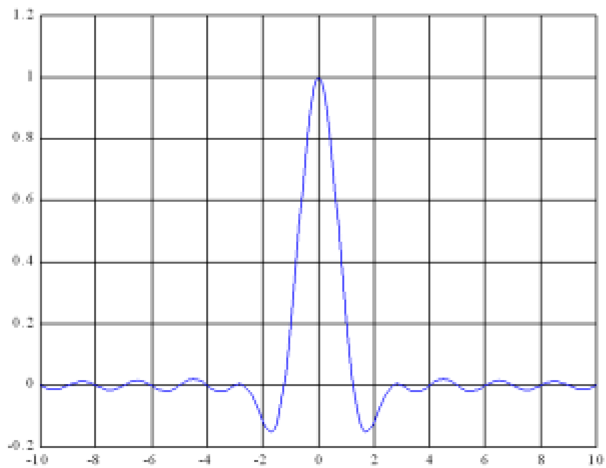

Figure 5: Inter-symbol Interference in q(t)

4. CONCLUSION

The concept of a receiver SNR facilitates evaluation of the performance of data transmission systems with various compensation methods or equalizer for Inter-symbol Interference. Use of SNR as a performance measure builds upon the simplifications often directly related to probability of error and is a function of both the receiver and the decision regions for the SBS detector. There are several inter-symbol interference and equalization methods, which amount to different structures for the receiver. An alternative suboptimal receiver can detect each of the successive *K* messages independently with no loss in performance incurred via the matched-filter/sampler. The symbol-by-symbol (SBS) detector, while optimum for the AWGN channel, will not be a maximum-likelihood estimator for the sequence of messages. The bank of matched filters, found by Gram-Schmitt decomposition of the set of channel output waveforms, precedes a block detector that determines the *K*-dimensional vector symbol transmitted. This reduces the determinant complexity for the block detector.

References

.Kaur, R. and Sonia Goyal, S. (2013), Bit Error Rate Reduction in MIMO Systems Using Equalization Techniques, International Journal of Engineering Science and Innovative Technology, Vol. 2, Issue 3, pp 36-40.

Orlic, V. D and Lutovac, M. (2009), A Solution for Efficient Reduction of Inter-symbol Interference in Digital Microwave Radio, Microwave Review.

Singal, T. L. (2011), Mitigation of Inter-symbol Interference with Equalization and Convolution Coding, International Journal of Computing and Business Research, Vol. 2, Issue 3.

Wong & Lok: Theory of Digital Communications: Inter-symbol Interference and Equalization, http://wireless.ece.ufl.edu/twong/Notes/Comm/ch4.pdf.