On soft Heine-Borel theorem

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Abstract. In this paper we define soft member , soft bounded set in soft real line , soft usual topology , finite soft set , soft cofinite topology , introduce soft Heine-Borel theorem and converse of soft Heine-Borel theorem . And study the conditions that make the soft set be a soft compact set explained with examples.

Keywords: soft member, soft bounded set, soft locally compact, soft usual topology, soft cofinite topology, and soft Heine-Borel theorem.

1 INTRODUCTION

A soft set was introduced by Demetry Molodtsove 1999 as a general mathematical tool for dealing with uncertain objects. P.K. Maji, R. Biswas, R. Roy 2003define soft union and soft subset, F. Feng, Y.B. Jun, X.Z. Zhao 2008define soft intersection, M.Sabir and M.Nas 2011introduce and study the concept of soft topological spaces over soft set and some related concepts such as soft Hausdorff space, E.Peyghan, B.Samadi, A.Tayebi 2013 define soft relative topology, Zorltuna, MAkdag, W.K. Min, S.Atmaca 2012 define soft compact space, Banu Pazar Varol,Halis Aygun 2013worked on soft Hausdorff spaces , Dariusz Wardowski in 2013introduce soft element. In this paper we will, soft usual topology , soft cofinite topology soft Heine-Borel theorem, converse of soft Heine-Borel theorem ,soft Weierstrass theorem And study under what conditions the soft set and a soft topological space became a soft compact set , soft compact space respectively , declared with examples .

2 PRELIMINARIES

2.1 Soft sets and soft topological spaces

In this section we study a basic definitions and properties on soft set **,** where X be an initial universe set and E set of parameters, P(X) denote the power set of X ,$SS(X)_E$ denote to the set of all soft sets (F,E) over a universe X in which all parameter E are the same **,** through this paper all soft sets belong to $SS(X)_E$.

2.1.1 Definition Let X be an initial universe and E be a set of parameters, P(X) denote the power set of X , A pair (F,E) is called a soft set over X , where F is a valued map given by
F: E$\rightarrow$ P(X) .

2.1.2 Definition For two soft sets (F,A) and (G,B) over common universe X , we say that (F,A) is a soft subset of (G,B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$, denoted by
$(F,A) \tilde{\subseteq} (G,B)$.

2.1.3 Definition The union of two soft sets of (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C= A\cup B$, and $\forall$ $e \in C$, we write
$(F,A) \tilde{\cup} (G,B)= (H,C)$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

2.1.4 Definition The intersection of two soft sets (F,A) and (G,B) over a common universe X is the soft set (H,C), where $C = A \cap B$, and , $H(e) = F(e) \cap G(e)$,We write
$(F,A) \tilde{\cap} (G,B) =(H,C) \forall e \in C$.

2.1.5 Definition A soft set is said to be null soft set denoted by $\tilde{\Phi}$, if $\forall e \in E$, $F(e) = \Phi$.
And a soft set is said to be an absolute soft set denoted by $\tilde{X}$, if $\forall e \in E$, $F(e) = X$.

2.1.6 Definition The soft difference (H,E) of two soft sets (F,E) and (G,E) over X denoted by $(F,E) \tilde{\setminus} (G,E)$ is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

2.1.7 Definition Let$(F,E) \tilde{\in} SS(X)_E$,a soft complement of (F,E), denoted by $(F,E)^c$ is a soft set of the form $(F,E)^c = \{ (e , X \setminus F(e)) : \forall e \in E \}$.

2.1.8 Definition Let $\tilde{T}$ be the collection of soft sets over X then $\tilde{T}$ is soft topology on X if.
1- $\tilde{\Phi}, \tilde{X}$ Belong to $\tilde{T}$.
2- The union of any number of soft sets in $\tilde{T}$ belongs to $\tilde{T}$.
3- The intersection of any two soft sets in $\tilde{T}$ belongs to $\tilde{T}$.
The triple $(X,\tilde{T},E)$ is called a soft topological space over X denoted by (S.T.S.) .

2.1.9 Definition Let $(X,\tilde{T},E)$ be a S.T.S over X then the members of $\tilde{T}$ are said to be soft open sets in X their soft complements are called soft closed sets.

2.1.10 Definition Let $(X,\tilde{\boldsymbol{T}},E)$ be a S.T.S. over X ,(F,E) be a soft set over X and $x \in X$, then (F,E) is said to be a soft neighborhood of x if there exist a soft open set (G,E) such that
$\tilde{\boldsymbol{x}} \tilde{\in} (G,E) \tilde{\subseteq} (F,E)$, a soft neighborhood (F,E) of a soft element $\tilde{\boldsymbol{x}}$ will be denoted by $(\boldsymbol{F},\boldsymbol{E})_{\tilde{\boldsymbol{x}}}$.

2.1.11 Definition Let (F,E) be a soft set over X,Y be a non empty subset of X then the soft subset (F,E) over Y denoted by $(^{Y}F,E)$ is defined as follows
$^{Y}F(e) = Y \cap F(e)$ for each $e \in E$ (in other word $(^{Y}F, E) = \tilde{Y} \tilde{\cap} (F,E)$.

2.1.12 Definition Let $(X,\tilde{T},E)$ be soft topological space over X and Y be a non empty subset of X , then $\tilde{T}_Y =\{(^{Y}F,E) | (F,E) \in \tilde{T}\}$is said to be the soft relative topology on Y and $(Y,\tilde{T}_Y, E)$ is called soft subspace topology of $(X,\tilde{T},E)$.

2.2 Soft bounded of a soft set

2.2.1 Definition Let $(F,E) \tilde{\in} SS(X)_E$ we say that $\tilde{x} =(e,\{h\})$is a non empty soft element of (F,E) if $e \in E$ and $h \in F(e)$. The pair (e,Φ) is called an empty soft element of (F,E) .Non empty soft elements of (F,E) and empty soft elements of (F,E) are called the soft elements of (F,E).The fact that $\tilde{x}$ is a soft element of (F,E) will be denoted by $\tilde{x} \tilde{\in} (F,E)$.

2.2.2 Definition Let $(F,E) \tilde{\in} SS(X)_E$,let $\tilde{x} =(e,\{h\})$ be a non empty soft element of (F,E) if $e \in E$ and $h \in F(e)$and if $x \in X=IR$, then $\tilde{x}$ is called soft real element .

2.2.3 Definition Let X be a non empty set and E be a non empty set of parameters ,
$(F,E) \tilde{\in} SS(X)$. We say that the $(e,F(e)) \forall e \in E$ be the soft members of the soft set (F,E) denoted by $\tilde{\mu}$ and $\tilde{\mu} \tilde{\in} (F,E)$.

2.2.4 Example Let $X =\{h_1, h_2, h_3\}$, $E=\{e_1,e_2\}$. Take a soft set $(F,E) \tilde{\in} SS(X)_E$ of the form $(F,E) =\{(e_1,\{ h_1\}),\{(e_2,\{h_1,h_2\})\}$. Then all the soft elements of (F,E) are the following: (e_1,Φ), $(e_1,\{h_1\})$, (e_2, Φ),$(e_2,\{h_1\})$, $(e_2,\{h_2\})$ .if $e_1=h_1=5$ then $(e_1,\{ h_1\})=(5,\{5\})=\tilde{5}$ is soft real element .
The soft members of (F,E) are $(e_1,\{ h_1\})$,$(e_2,\{h_1,h_2\})$.

2.2.5 Definition
Let X=E=IR , a soft set of soft real numbers defined by
$\tilde{\mathfrak{R}} = \{ \ldots,-\tilde{1}, \ldots , \tilde{0}, \ldots , \tilde{1},\ldots \}$ each soft member $\tilde{\mu}$ of the soft set $\tilde{\mathfrak{R}}$ defined as follows :
$\tilde{\mu} = (e,F(e))$, $F(e) =e$, $\forall e \in E$.

2.2.6 Notifications

1. If $\tilde{x}$, $\tilde{\varepsilon}$ are soft real numbers then :
   $\tilde{x} \tilde{+} \tilde{\varepsilon} = (e_1,F(e_1)) \tilde{+} (e_2,F(e_2)) = (e_1,+ e_2 , F(e_1)+F(e_2))= \tilde{a}$,
   $\tilde{x} \tilde{-} \tilde{\varepsilon} = (e_1,F(e_1)) \tilde{-} (e_2,F(e_2)) = (e_1- e_2 , F(e_1)-F(e_2)) = \tilde{b}$.
2. Let $\tilde{\varepsilon}$ is the smallest soft positive real number then $(F,E)_{\tilde{x}} = (\tilde{x} \tilde{-} \tilde{\varepsilon} , \tilde{x} \tilde{+} \tilde{\varepsilon})$ is a soft open interval in the soft set $\tilde{\mathfrak{R}}$ which contain the soft element $\tilde{x}$.

3. $(\tilde{x}, \tilde{y}) = \{ \tilde{a} | \ \tilde{x} \ \tilde{<} \ \tilde{a} \ \tilde{<} \ \tilde{y} : \tilde{a}, \tilde{x}, \tilde{y} \ \tilde{\in} \ \tilde{\Re}\}$ is soft open interval in the soft set $\tilde{\Re}$.
4. $[\tilde{x}, \tilde{y}] = \{ \tilde{a} | \ \tilde{x} \ \tilde{\leq} \ \tilde{a} \ \tilde{\leq} \ \tilde{y} : \tilde{a}, \tilde{x}, \tilde{y} \ \tilde{\in} \ \tilde{\Re}\}$ is soft close interval in the soft set $\tilde{\Re}$.
5. $(\tilde{x}, \tilde{y}] = \{ \tilde{a} | \ \tilde{x} \ \tilde{<} \ \tilde{a} \ \tilde{\leq} \ \tilde{y} : \tilde{a}, \tilde{x}, \tilde{y} \ \tilde{\in} \ \tilde{\Re}\}$ is soft half open interval in the soft set $\tilde{\Re}$.
6. $[\tilde{x}, \tilde{y}) = \{ \tilde{a} | \ \tilde{x} \ \tilde{\leq} \ \tilde{a} \ \tilde{<} \ \tilde{y} : \tilde{a}, \tilde{x}, \tilde{y} \ \tilde{\in} \ \tilde{\Re}\}$ is soft half close interval in the soft set $\tilde{\Re}$.

2.2.7 Definition Consider (F,E) be a soft set in the soft real numbers $\tilde{\Re}$, (F,E) is called soft bounded above if there is a soft number $\tilde{n}$ so that any $\tilde{x} \ \tilde{\in}$ (F,E) is soft less than, or soft equal to $\tilde{n}$: $\tilde{x} \ \tilde{\leq} \ \tilde{n}$. The soft number $\tilde{n}$ is called a soft upper bound for the soft set (F,E) .

2.2.8 Definition A soft number $\tilde{t}$ is called the soft least upper bound (or soft supremum) of the soft set (F,E) if:

1. $\tilde{t}$ is a soft upper bound: any $\tilde{x} \ \tilde{\in}$ (F,E) satisfy's $\tilde{x} \ \tilde{\leq} \ \tilde{t}$, and
2. $\tilde{t}$ is the smallest soft upper bound i.e. If $\tilde{k}$ is any other soft upper bound of the soft set (F,E) then $\tilde{t} \ \tilde{<} \ \tilde{k}$.

2.2.9 Definition A soft set (F,E) is called soft bounded below if there is a soft number $\tilde{m}$ so that any $\tilde{x} \in$ (F,E) is soft bigger than, or soft equal to $\tilde{m}$: $\tilde{x} \ \tilde{\geq} \ \tilde{m}$. The soft number $\tilde{m}$ is called a soft lower bound for the soft set (F,E) .

2.2.10 Definition A soft number $\tilde{s}$ is called the soft greatest lower bound (or soft infimum) of the soft set (F,E) if :

1. $\tilde{s}$ is a soft lower bound : any $\tilde{x} \ \tilde{\in}$ (F,E) satisfies $\tilde{s} \ \tilde{\leq} \ \tilde{x}$, and
2. $\tilde{s}$ is the greatest soft lower bound i.e. If $\tilde{k}$ is any other soft lower bound of the soft set (F,E) then $\tilde{k} \ \tilde{<} \ \tilde{s}$.

2.2.11 Definitions

1. When the soft supremum of a soft set (F,E) is a soft number that soft belong to (F,E) then it is also called soft maximum .
2. When the soft infimum of a soft set (F,E) is a soft number that soft belong to (F,E) then it is also called soft minimum .

2.2.12 Examples

1. a soft set $(\tilde{1}, \tilde{5})$ soft infimum = $\tilde{1}$ and soft supremum = $\tilde{5}$ **,** no soft minimum or soft maximum are exist**.**
2. In a soft set $[\tilde{1}, \tilde{5}]$ the soft minimum = soft infimum = $\tilde{1}$ and

soft maximum = soft supremum = $\tilde{5}$.

2.2.13 Definition A soft set which is soft bounded above and soft bounded below is called soft bounded. So if (S,E) is a soft bounded set then there are two soft numbers, $\tilde{m}$ and $\tilde{n}$ so that $\tilde{m} \ \tilde{\leq} \ \tilde{x} \ \tilde{\leq} \ \tilde{n}$ for any $\tilde{x} \ \tilde{\in}$ (S,E) . A set which is not soft bounded is called soft unbounded.

2.2.14 Example the soft set of soft real numbers $\tilde{\Re}$ is soft unbounded **.**

3 MAIN RESULTS

3.1 Soft usual topology and soft cofinite topology

3.1.1 Definition A soft set (F,E) is said to be (finite soft set) iff the set of parameters E and $F(e_i)$,$\forall$ i $\in \xi$ are finite sets.

3.1.2 Definition A family Ω of soft sets is a (soft cover) of a soft set (F,E) if $(F,E) \ \tilde{\subseteq} \ \tilde{\cup}\{(F,E)_\lambda : (F,E)_\lambda \in \Omega, \lambda \in \xi \}$.It is a (soft open cover) if each member of Ω is a soft open set . A (soft subcover) of Ω is a subfamily of Ω which is also a cover.

3.1.3 Definition A soft topological space $(X,\tilde{T},E)$ is (soft locally compact) iff each soft point of $\tilde{X}$ is contained in a soft compact neighborhood .

3.1.4 Definition A soft topological space $(X,\tilde{T},E)$ is (soft compact) if each soft open cover of $\tilde{X}$ has a finite subcover.

3.1.5 Lemma Let $(X,\widetilde{T},E)$ be a soft topological space. A soft subset (F,E) of $(X,\widetilde{T},E)$ is soft compact (with respect to the soft relative topology on (F,E)) iff for each soft open cover Ω of soft open sets in $(X,\widetilde{T},E)$ covering soft set (F,E) there exists a finite soft subcover .

Proof A soft subset (G,E)* of (F,E) is soft open in (F,E) (with respect to the relative topology on (F,E)) if and only if $(G,E)^* = (F,E) \widetilde{\cap} (G,E)$ for some soft open set (G,E) in $(X,\widetilde{T},E)$. The result follows directly from the definition of soft compactness.

3.1.6 Remark Since $\widetilde{X}$ is always a finite soft neighborhood of each of its soft points then every soft compact space is soft locally compact but the converse is not necessary true.

3.1.7 Example The soft indiscrete space is soft compact and soft locally compact but soft discrete topology defined on infinite set is soft locally compact but not soft compact.

3.1.8 Proposition Every soft closed soft subset of soft locally compact space is soft locally compact .

Proof Let (F,E) be a soft closed soft subset of the soft locally compact space $(X,\widetilde{T},E)$. Let $\tilde{x}$ be a soft element of (F,E). Then $\tilde{x}$ is a soft element of $(X,\widetilde{T},E)$, and therefore has a soft compact neighbourhood (C,E). Then the intersection $(C,E) \widetilde{\cap} (F,E) = (M,E)$ is soft closed in the soft space (C,E), which is soft compact, so (M,E) is soft compact. Since $\tilde{x}$ is a soft element of (M,E) , so $\tilde{x}$ has a soft compact neighbourhood , and since $\tilde{x}$ in (F,E) was arbitrary, this shows (F,E) is soft locally compact.

3.1.9 Proposition Every soft closed soft subset of soft compact space is soft compact.

Proof Let $\{(G,E)_\lambda\}$ be a soft open cover for (F,E) then $\{\widetilde{\cup}_\lambda (G,E)_\lambda \widetilde{\cup} (F,E)^c\}$ is soft open cover for $\widetilde{X}$, since $\widetilde{X}$ is soft compact then there exist a soft finite subcover, $\widetilde{X} = [\widetilde{\cup}_{i=1}^{n} (G,E)_{\lambda i}] \widetilde{\cup} (F,E)^c$ therefore $(F,E) \widetilde{\subseteq} \widetilde{\cup}_{i=1}^{n} (G,E)_{\lambda i}$ so (F,E) is soft compact .

3.1.10 Definition Let $(X,\widetilde{\boldsymbol{T}},E)$ be a soft topological space and (F,E) be a soft set then int $(F,E)= \widetilde{\cup}\{(O,E) : (O,E)$ is soft open and $(O,E) \widetilde{\subseteq} (F,E)\}$(i.e. int (F,E) is the largest soft open set contained in (F,E)) .

3.1.11 Proposition [M. Shabir and M. Naz (2011)] Let (F,E), (G,E) be two soft subsets of a S.T.S. $(X,\widetilde{\boldsymbol{T}},E)$ then $\mathrm{int}[(F,E) \widetilde{\cap} (G,E)] = \mathrm{int}(F,E) \widetilde{\cap} \mathrm{int}(G,E)$.

3.1.12 Theorem Let X = E = IR , $\widetilde{T}_u =\{(F,E) \mid (F,E) = (\tilde{x}, \tilde{y}) : \tilde{x} , \tilde{y}$ are soft elements in the soft set of real numbers $\widetilde{\Re}\}\widetilde{\cup}\{\widetilde{\Phi},\widetilde{\Re}\}$ then $\widetilde{T}_u$ is soft usual topology on IR **.**

Proof

1- $\widetilde{\Phi},\widetilde{\Re} \;\widetilde{\in}\; \widetilde{T}_u$

2- Let $(F,E)_\lambda \widetilde{\in} \widetilde{T}_u , \forall \lambda \in \xi$ if $\widetilde{\cup}_\lambda (F,E)_\lambda = \widetilde{X}$ then there is nothing to prove

Let $\tilde{x} \;\widetilde{\in}\; \widetilde{\cup}_\lambda (F,E)_\lambda$ then $\tilde{x} \;\widetilde{\in}\; (F,E)_\lambda$ for some $\lambda \in \xi$

$\tilde{x} \;\widetilde{\in}\; (F,E)_\lambda \widetilde{\subseteq} \widetilde{\cup}_\lambda (F,E)_\lambda$,hence $\widetilde{\cup}_\lambda (F,E)_\lambda \widetilde{\in} \widetilde{T}_u$

3- Let (F,E) , (G,E) $\widetilde{\in} \widetilde{T}_u$, then they are soft open hence they are equal to their soft interior then by Proposition **(**3.1.11**)** $(F,E) \widetilde{\cap} (G,E) = \mathrm{int}\,(F,E) \widetilde{\cap} \mathrm{int}\,(G,E) = \mathrm{int}\,[(F,E) \widetilde{\cap} (G,E)]$ hence $(F,E) \widetilde{\cap} (G,E) \widetilde{\in} \widetilde{T}_u$.

3.1.13 Definition Let X=E=IR ,Then $\widetilde{T}_u =\{(F,E) \mid (F,E) = (\tilde{x}, \tilde{y})$ and $\tilde{x}, \tilde{y}$ are soft element in the soft set of real numbers $\widetilde{\Re}\}\widetilde{\cup}\{\widetilde{\Phi},\widetilde{\Re}\}$ then $\widetilde{T}_u$ is called (soft usual topology) on IR and $(IR,\widetilde{T}_u,E)$ is called (soft usual topological space) on IR .

3.1.14 Definition A soft closed set (F,E) in $(IR,\widetilde{\boldsymbol{T}}_u,E)$ defined as follows:

Let X=E=IR , $(F,E) =\{[\tilde{x}, \tilde{y}] \mid \tilde{x}, \tilde{y}$ are soft element in the soft set of real numbers $\widetilde{\Re}\}$.

3.1.15 Examples

1. Let X = E = IR , Then $\widetilde{T} = \{\widetilde{\Phi}, \widetilde{\Re},(\tilde{1}, \tilde{2})\}$,$(IR,\widetilde{T},E)$ is soft compact space on IR but not soft usual space.
2. soft usual space is not soft compact space .

3.1.16 Proposition A soft compact subset of $(R,\widetilde{T}_u,E)$ is soft bounded.

Proof Let (F,E) be unbounded soft compact subset of $(R,\widetilde{T}_u,E)$ then $(F,E) \widetilde{\subseteq} \widetilde{\cup}_{i=1}^{\infty}(-\tilde{n}, \tilde{n})$

but { (-ñ, ñ): ñ = $\tilde{1}, \tilde{2},\ldots$} does not have a finite soft subcover of (F,E) which is c! since (F,E) is soft compact hence a soft compact subset of (IR,$\tilde{T}_u$,E) must be soft bounded.

3.1.17 Theorem Let $X \neq \tilde{\Phi}$, $\tilde{T}_c = \{(F,E) \tilde{\subseteq} \tilde{X}: (F,E)^c$ is finite soft set$\} \tilde{\cup} \tilde{\Phi}$

Then $\tilde{T}_c$ is soft cofinite topology on X.

Proof (i) $\tilde{\Phi} \tilde{\in} \tilde{T}_c$ by definition, since $\tilde{X}^c = \tilde{\Phi}$ is finite soft set then $\tilde{X} \tilde{\in} \tilde{T}_c$.

(ii) Let $(G,E)_\lambda \tilde{\in} \tilde{T}_c$ $\forall \lambda \in \xi$ then $(G,E)_\lambda^c$ are finite soft open sets

Since $\tilde{\cap}_\lambda [(G,E)_\lambda]^c = [\tilde{\cup}_\lambda (G,E)_\lambda]^c$

And the soft intersection of finite soft open sets is finite soft set then

$\tilde{\cap}_\lambda[(G,E)_\lambda]^c$ is finite hence $\tilde{\cup}_\lambda(G,E)_\lambda \tilde{\in} \tilde{T}_c$.

(iii) Let $(G,E)_i \tilde{\in} \tilde{T}_c$, $i \in \zeta$, ζ finite set then $(G,E)_i^c$ is soft finite sets for each i=1,2,…n

And the soft union of finite soft open sets is finite soft set then

$\tilde{\cup}_{i=1}^n [(G,E)_i]^c = [\tilde{\cap}_{i=1}^n (G,E)_i]^c$ is finite soft set, hence $\tilde{\cap}_{i=1}^n (G,E)_i \tilde{\in} \tilde{T}_c$.

3.1.18 Definition Let $X \neq \tilde{\Phi}$, $\tilde{T}_c = \{(F,E) \tilde{\subseteq} \tilde{X}: (F,E)^c$ finite soft set$\} \tilde{\cup} \tilde{\Phi}$ is soft cofinite topology, (X,$\tilde{T}_c$,E) is called soft cofinite topological space.

3.1.19 Example Let X={h_1,h_2}, E={e_1,e_2} Then all soft sets of $\tilde{X}$ s.t. their relative complements are finite sot set are:

$(F,E)_1 = \tilde{X} \tilde{\subseteq} \tilde{X}$: $\tilde{X}^c = \tilde{\Phi}$

$(F,E)_2 = \{\{h_1\}, \Phi\} \tilde{\subseteq} \tilde{X}$: $(F,E)_2^c = \{\{h_2, h_3\}, X\}$ is finite soft set

$(F,E)_3 = \{\{h_2\}, \Phi\} \tilde{\subseteq} \tilde{X}$: $(F,E)_3^c = \{\{h_1, h_3\}, X\}$ is finite soft set

$(F,E)_4 = \{X,\Phi\} \tilde{\subseteq} \tilde{X}$: $(F,E)_4^c = \{\Phi, X\}$ is finite soft set

Similarly $(F,E)_5 = \{\Phi,\{h_1\}\}$, $(F,E)_6 = \{\Phi, \{h_2\}\}$, $(F,E)_7 = \{\Phi, X\}$

Then $\tilde{T}_c = \{(F,E)_i: (F,E)_i^c$ is finite soft set$\} \tilde{\cup} \tilde{\Phi}$ for each i = 1, … , 7

Notice that the soft open neighborhoods' for the soft element (e_1,{h_1}) are

$\{\{h_1, h_3\},X\},\{\{h_1\},\Phi\},\{X,\Phi\}, \tilde{X}$, and for the soft element (e_2,{h_2}) are

$\tilde{X}$, $\{\Phi, \{h_2\}\}$, $\{\Phi,X\}$.

3.1.20 Proposition The soft cofinite topology (X,$\tilde{T}_c$,E) is soft compact.

Proof Let $\{(G,E)_\lambda\}$ be soft open cover for $\tilde{X}$, let $(G,E)_o \tilde{\in} \{(G,E)_\lambda\}$since $(G,E)_o \tilde{\in} \tilde{T}_c$ then $(G,E)_o^c$ is finite soft set . Then for each soft element $\tilde{x} \tilde{\in} (G,E)_o^c$, $\exists (G,E)_{\lambda i} \tilde{\in} \{(G,E)_\lambda\}$ s.t. $\tilde{x} \tilde{\in} (G,E)_{\lambda i}$, hence $(G,E)_o^c \tilde{\subseteq} (G,E)_{\lambda 1} \tilde{\cup} (G,E)_{\lambda 2} \tilde{\cup} \ldots \tilde{\cup} (G,E)_{\lambda n}$

and $\tilde{X} = (G,E)_o \tilde{\cup} (G,E)_o^c = (G,E)_o \tilde{\cup} (G,E)_{\lambda 1} \tilde{\cup} \ldots \tilde{\cup} (G,E)_{\lambda n}$, thus (X,$\tilde{T}_c$,E) is soft compact.

3.1.21 Definition Let (X,$\tilde{T}$,E) be a soft topological space and $\tilde{x}, \tilde{y}$ are two different soft elements in (X,$\tilde{T}$,E) then the S.T.S. is called (soft Hausdorff space or $\tilde{T}_2$ space) if there exist soft open sets (G,E),(H,E) such that $\tilde{x} \tilde{\in} (G,E)$, $\tilde{y} \tilde{\in} (H,E)$ and $(G,E) \tilde{\cap} (H,E) = \tilde{\Phi}$.

3.1.22 Example Any soft discrete space is soft Hausdorff space.

3.1.23 Proposition **[**Dariusz Wardowski (2013)] Every soft compact soft subset (F,E) of a soft Hausdorff space (X,$\tilde{T}$,E) is soft closed set .

3.1.24 Proposition Let (X,$\tilde{T}$,E) be any soft topological space and (F,E) be a finite soft subset of (X,$\tilde{T}$,E) then (F,E) is soft compact .

Proof Let $\{(G,E)_\lambda\}$, $\lambda \in \xi$, be any family of soft open sets such that $(F,E) \tilde{\subseteq} \tilde{\cup}_\lambda (G,E)_\lambda$. Then for each soft element $\tilde{x}_j \tilde{\in} (F,E)$, there exists an $(G,E)_{\lambda j}$, such that $\tilde{x}_j \tilde{\in} (G,E)_{\lambda j}$. Thus $(F,E) \tilde{\subseteq} (G,E)_{\lambda 1} \tilde{\cup} (G,E)_{\lambda 2} \tilde{\cup} \ldots \tilde{\cup} (G,E)_{\lambda n}$ then (F,E) is soft compact .

now we give a new conditions that make a soft set be soft compact set and a soft topological space be soft compact space .

3.1.25 Proposition In a soft set (F,E) If the sets F(e_i) are compact $\forall e_i \in E$, E is finite set then the soft set (F,E) is soft compact set in a S.T.S. (X,$\tilde{T}$,E) .

Proof Obvious .

3.1.26 Examples

1. Let (X,$\tilde{T}$,E) be soft topological space X= IR , E={e_1,e_2,e_3}

Let $\tilde{T} = \{\tilde{\Phi}, \tilde{X}, (F,E)\}$,(F,E)={($e_1$,F($e_1$)),($e_2$,F($e_2$)),($e_3$,F($e_3$))}where F($e_1$)= [1,2] , F($e_2$)= [2,3] ,

$F(e_3)= [4,5]$ then F(ei) are compact sets $\forall i= 1,2,3$, therefore (F,E) is soft compact set by Proposition (3.1.25) **.**

2. Let $(X,\widetilde{T},E)$ be soft topological space X = IR , E = IN , $\widetilde{T} =\{\widetilde{\Phi} , \widetilde{X} , (F,E)\}$ let $(F,E) = \{(e_1,F(e_1)),(e_2,F(e_2)),(e_3,F(e_3)\}, \dots \}$. Where $F(e_i) = [a,b]$, $a,b \in IR$ then F(ei) are compact sets $\forall$ i$\in$ IN, E is infinite , therefore (F,E) is not soft compact set .

3. Let $(X,\widetilde{T},E)$ be soft topological space X= IR , $E=\{e_1,e_2\}$, $\widetilde{T} =\{\widetilde{\Phi} , \widetilde{X} , (F,E)\}$ let $(F,E) = \{(e_1,F(e_1)),(e_2,F(e_2))\}$where $F(e_1) = [1,2]$,$F(e_2) = (4,5)$ then $F(e_1)$ is compact but not $F(e_2)$, therefore (F,E) is not soft compact set.

3.1.27 Remark The converse of proposition (3.1.25) is not true in general i.e.(if (F,E) is soft compact set then $\forall$ F(e) $\widetilde{\subseteq}$ (F,E) ,F(e) is not necessary compact set or E is not necessary finite set).

3.1.28 Example In a S.T.S. $(IR,\widetilde{T}_i,E)$ let E infinite set then $\widetilde{IR}$ is soft compact set since $\widetilde{IR}$ has only one cover which is $\widetilde{IR}$ itself but E infinite and F(ei) not compact $\forall$i .

As a special case of soft compactness of a S.T.S. we get the following results:

3.1.29 Remarks

1. In a soft topology $(X,\widetilde{T},E)$ if the sets E, X are finite then $(X,\widetilde{T},E)$ is soft compact space .
2. If the soft sets $(F,E)_i$ in a S.T.S. $(X,\widetilde{T},E)$ are finite (i.e. Fi are finite maps) then $(X,\widetilde{T},E)$ is soft compact space .

3.1.30 Examples

1. Let $X= \{h_1, h_2\}$, $E=\{e_1,e_2\}$, $\widetilde{T}=\{\widetilde{\Phi} , \widetilde{X} ,\{(e_1,\{h_1\}),(e_2,\{ h_2\})\}, \dots\dots\}$.
Where $(F,E)_1= \widetilde{\Phi}$, $(F,E)_2 = \widetilde{X}$, $(F,E)_i = \{(e_1,\{h_1\}),(e_2,\{ h_2\})\}\forall i = 3 ,\dots.. \infty$
then $(X,\widetilde{T},E)$ is soft compact space . let $\widetilde{X} = \{(e_1, X),(e_2,X)\} \widetilde{\subseteq} \widetilde{\cup}_\lambda (F,E)_\lambda$ then
$\widetilde{X} = \{(e_1, X),(e_2,X)\} \widetilde{\subseteq} (F,E)_2 \widetilde{\cup} (F,E)_3$ which is finite sub cover of $\widetilde{\cup}_\lambda (F,E)_\lambda$.
2. Let $(X,\widetilde{T},E)$ be soft topological space $X= \{h_1, h_2\}$, $E=\{e_1,e_2,e_3\}$
E , X are finite , $\widetilde{T}=\{\widetilde{\Phi}, \widetilde{X} ,\{(e_1,\{h_1\}),(e_2,\{ h_2\}) ,(e_3,\{h_1, h_2\})\}\}$
$\widetilde{X}=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1, h_2\}), (e_3,\{h_1, h_2\})\}$. Then $(X,\widetilde{T},E)$ is soft compact space .
3. Let $(X,\widetilde{T},E)$ be soft topological space X= [0,5] , $E=\{e_1,e_2,e_3\}$
$\widetilde{T}=\{\widetilde{\Phi} , \widetilde{X} ,((e_1,[0,1]),(e_2,[0,2]), (e_3,[0,3]))\}$, $\widetilde{X}=\{(e_1,[0,5]) ,(e_2,[0,5]), (e_3,[0,5])\}$
Then $(F,E)_i$ are finite .Therefore $(X,\widetilde{T},E)$ is soft compact space .

3.1.31 Remark The converse of remarks (**(**3.1.29) (1) , (2)) are not necessary true .

3.1.32 Examples

1. A S.T.S. $(IR,\widetilde{T}_i,E)$ is soft compact space since $\widetilde{IR}$ has only one cover which is $\widetilde{IR}$ itself But IR infinite and E are not necessary finite .
2. The soft cofinite topology is always soft compact even if the soft sets $(F,E)_i$ in a S.T.S. $(X,\widetilde{T}_c,E)$ are infinite .

3.1.33 Remark

In a S.T.S. $(X,\widetilde{T},E)$ if each F(e) is compact set and E is finite set then each (F,E) in $\widetilde{T}$ is soft compact set (by proposition (3.1.25)). And $(X,\widetilde{T},E)$ is soft compact space .

3.2 On soft Heine-Borel theorem

3.2.1 Theorem (Soft Heine-Borel)

A soft closed soft bounded soft interval in the soft usual topology is soft compact.

Proof Let $(IR,\widetilde{T}_u,E)$ be the soft usual topology on IR , Let $[\tilde{a},\tilde{b}]$ be a soft closed soft bounded soft interval in the soft real line with the soft relative topology and let $\wp$ be a soft open cover of $[\tilde{a},\tilde{b}]$. We define a soft subset $\widetilde{X}$ of $[\tilde{a},\tilde{b}]$ by $\widetilde{X} =\{ \tilde{x} \widetilde{\in} [\tilde{a},\tilde{b}] \mid [\tilde{a},\tilde{x}]$ is contained in the soft union of a soft finite subfamily of $\wp\}$. Then $\widetilde{X}$ is non- null soft set ($\tilde{a} \widetilde{\in} \widetilde{X}$ and is soft bounded above by $\tilde{b}$. So $\widetilde{X}$ has a soft supremum or soft least upper bound say $\tilde{s}$. We claim that $\tilde{s} \widetilde{\in} \widetilde{X}$ and that $\tilde{s} = \tilde{b}$ .For let (O,E) be the soft member of $\wp$ which contains $\tilde{s}$. since (O,E) is soft open we can choose $\tilde{\varepsilon} \widetilde{>} \tilde{0}$ small enough that $(\tilde{s} - \tilde{\varepsilon} , s] \widetilde{\subseteq}$ (O,E), and if $\tilde{s}$ is less than $\tilde{b}$ we can assume $(\tilde{s} - \tilde{\varepsilon} , \tilde{s} + \tilde{\varepsilon}] \widetilde{\subseteq}$ (O,E) . Now $\tilde{s}$ is the soft least upper bound of $\widetilde{X}$, consequently there are soft elements of $\tilde{X}$ arbitrarily close to $\tilde{s}$. Also $\tilde{X}$ has the property that if $\tilde{x} \tilde{\in} \tilde{X}$ and if $\tilde{a} \lesseqq \tilde{y} \lesseqq \tilde{x}$ then $\tilde{y} \tilde{\in} \tilde{X}$. Therefore we may assume $\tilde{s} - \frac{\tilde{\varepsilon}}{2} \tilde{\in} \tilde{X}$. by the definition of $\tilde{X}$, the soft interval $[\tilde{a} , \tilde{s} - \frac{\tilde{\varepsilon}}{2}]$ is contained in the soft union of some soft finite subfamily $\wp^*$ of $\wp$. Adding (O,E) to $\wp^*$ we obtain a finite collection of soft members of $\wp$ whose union certainly contains $[\tilde{a}, \tilde{s}]$. Therefore $\tilde{s} \tilde{\in} \tilde{X}$. If $\tilde{s}$ is less than $\tilde{b}$ then $[\tilde{\cup} \wp^*] \tilde{\cup}$ (O,E) contains $[\tilde{a} , \tilde{s} + \frac{\tilde{\varepsilon}}{2}]$, giving $\tilde{s} + \frac{\tilde{\varepsilon}}{2} \tilde{\in} \tilde{X}$ and contradicting the fact that $\tilde{s}$ is a soft upper bound for $\tilde{X}$ .Therefore $\tilde{s} = \tilde{b}$ and all of $[\tilde{a}, \tilde{b}]$ is contained in $[\tilde{\cup} \wp^*] \tilde{\cup}$ (O,E) .

3.2.2 Theorem (Converse of Soft Heine-Borel theorem) Every soft compact soft subset in $(R,\tilde{T}_u,E)$ is soft closed and soft bounded .

Proof Let (F,E) be soft compact soft subset of $(R,\tilde{T}_u,E)$, by Propositions (3.1.16) (F,E) is soft bounded . And since $(R,\tilde{T}_u,E)$ is soft Hausdorff space then by Propositions (3.1.23**)** the soft set (F,E) is soft closed .

3.2.3 Definition A soft subset (F,E) of a S.T.S. $(X,\tilde{T},E)$ is said to be (soft isolated set) iff (F,E) $\tilde{\cap}$ d (F,E) = $\tilde{\Phi}$.

3.2.4 Example Let $X=\{h_1, h_2\}$, $E=\{e_1, e_2\}$, $\tilde{T}=\{\tilde{\Phi}, \tilde{X} ,\{\{h_1\},\Phi\}\}$ is soft topology on X , the soft set $\{\{h_1\}, \Phi\}$ is soft isolated .

3.2.5 Theorem (Soft Weierstrass theorem) In a soft compact topological space $(X,\tilde{T},E)$ every infinite soft subset has a soft accumulation points.

Proof We prove that if, for a soft subset (F,E) of $(X,\tilde{T},E)$, the soft set of soft accumulation points d(F,E) is a null soft set then (F,E) is finite . Since cl(F,E) = (F,E) $\tilde{\cup}$ d(F,E) .Therefore if d(F,E) = $\tilde{\Phi}$ Then cl(F,E) = (F,E) = (F,E) $\tilde{\setminus}$ d(F,E).

The soft set (F,E) $\tilde{\setminus}$ d(F,E) is the soft set of soft isolated elements of (F,E)

hence all soft elements of (F,E) are soft isolated : for every $\tilde{x} \tilde{\in}$ (F,E)

there exists $(G, E)_{\tilde{x}}$ soft open set such that $(G, E)_{\tilde{x}} \tilde{\cap}$ (F,E) $=\{\tilde{x}\}$.Then

$cl(F,E) = (F,E) \tilde{\subseteq} \tilde{\cup}_{\tilde{x} \tilde{\in} (F,E)}(G, E)_{\tilde{x}}$

The soft set cl(F,E) is soft closed set in a soft compact topological space, therefore it is soft compact. Hence $cl(F,E) = (F,E) \tilde{\subseteq} \tilde{\cup}_{i=1}^{n} (G, E)_{\tilde{x}i}$

Then $(F,E) = [\tilde{\cup}_{i=1}^{n} (G,E)_{xi}] \tilde{\cap} (F,E) = \tilde{\cup}_{i=1}^{n} \{\tilde{x}\, i\}= \{\tilde{x}_{1}, \tilde{x}_{2}, \ldots \tilde{x}_{n}\}$.

3.2.6 Definition A property P of a S.T.S.$(X,\tilde{T},E)$ is called (soft hereditary property) iff every soft subspace $(Y,\tilde{T},E)$ of $(X,\tilde{T},E)$ also possesses a property P.

3.2.7 Remarks

1. Soft locally compactness is soft hereditary property on soft close set see proposition (3.1.8) .
2. Soft compactness is not soft hereditary property in general .It is soft hereditary only on soft close set as proved in proposition (3.1.9) .

3.2.8Example

Let X=E=IR with the soft usual topology, $[\tilde{1}, \tilde{2}]$ be soft closed set with the soft relative topology by soft Heine Borel theorem it is soft compact set , the soft subset $(\tilde{1}, \tilde{2})$ of $[\tilde{1}, \tilde{2}]$ is soft open not soft close not soft compact but $[\tilde{1}.5, \tilde{1}.20]$ is soft compact .

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