# Mathematical Modeling and to carry out a prototype helpmate differential drive robot for hospital purpose 

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#### Abstract

Robots are quite adjustable with the human society. Especially in industries, hospitals, school-college, military in everywhere. But developing countries are much more behind with this. Food, cloths, shelter, education, health in every sector they are struggling. The authors choose one of the sectors of the basic needs, where robots can perform as a helpmate for hospitals or clinics. The helpmate uses wheel encoders and sensors information to steer. This paper presents mathematical modeling, hardware and electrical design and implementation of prototype differential drive helpmate robot, with a complete navigation system is aided by four types of behaviors which help to reach its destination successfully.


Keywords: Mobile robot, Helpmate, IR sensor, BBB.

## 1 INTRODUCTION

Mobile Robot is a robot which can move one place to another. Research on mobile robot has placed highly emphasis on perception, modeling and navigation. As a result, robots have been designed considering their limitations. Ultimately robots are limited less by planning and more by dynamic behavior. The simplest case of mobile robots is wheeled robots, which capable of dynamic behavior ( Xu and $\mathrm{Ou}, 2005$ ). Wheeled robots comprise one or more driven wheels and have optional caster wheel and possibly steered wheels. Most designs require two motors for driving a mobile robot (Braunl, 2006). According to the wheel configuration mobile robots can be classified as differential driven, Omnidirectional, car-like, Synchro-drive (Balkcom and Mason, 2002; Arvin et al., 2009; Song and Byun, 2006; Cho and Ryeu, 2006; Fox et al., 1996).

A helpmate differential drive prototype has been made which has two motors mounted in fixed position on the left and right side of the robot with one caster wheels (Braunl, 2006). The helpmate will able to carry the meal trays, sterile supplies, medicine, medical reports, records, and provided necessary information to the patient to update them. Odometry-based navigation system enhanced by infrared (IR) sensor, which able them to deal easily with cluttered, unstructured environments.

The helpmate robot is arranged with five infrared sensors. Two of them are side of the wheels and rests of them are in front of robot. The helpmate is powered by 8AA batteries with two motors and wheel encoders can be controlled via MATLAB on its embedded micro-computer, the BeagleBone Black (BBB).

The simulator emulates the helpmate, which is a timer object. The execute function inside the supervisor has been called at every time step. The supervisor gathers the sensor information from the robot. Then passes the sensor information to the execute function of the controller alone with an estimate of the robot's position and orientation. Linear and angular velocity of the robot has been computed by the execute function and then converted to the left and right speeds of the robot and passed back to the robot. Supervisor also updating the estimate of the helpmate's position in orientation and the whole process in repeated at every time step (De la Croix, 2014).

The Helpmate has been made such a way that doctor and nurse can reprogram it and interface with it very easily. Designed helpmate robot is a mechatronic system, which is not only a marriage of electrical and mechanical systems and is more than just a control system; it is a complete integration of all them (Bishop, 2006).

The paper is organized as follows. Section 2 presents the mathematical approach of differential wheel drive, wheel encoder, Odometry and IR Range Sensor. Section 3 introduces the transformation from robot coordinate frame to the world coordinate frame. Section 4 educates the navigation system of helpmate robot. Section 5 describes the mechanical and electrical design of the helpmate robot respectively. Section 6 concludes the paper.

## 2 HELPMATE KINEMATICS: A MATHAMATICAL APPROACH

Kinematics is the most basic study of mechanical system behavior. In order to design mobile robot and to understand the control system understanding of mechanical system behavior is necessary (Siegwart and Nourbakhsh, 2004).

### 2.1 Differential Drive Model

A differential drive robot has two wheels. The wheels can turn at different rates and by turning the wheels at different rates the robot moves around. To design a differential drive model two parameters are needed. First one is wheel base (L), which is the distance between two wheels and second one is radius of the wheel $(\mathrm{R})$, which meaning about the size of the wheel. Now, $V_{r}$ and $V_{1}$ are two inputs. They are the rate at which the right and left wheel are turning respectively. $\mathrm{x}, \mathrm{y}$ and $\theta$ is the position and orientation of the helpmate respectively. After transition between inputs and states the resulting equations are

$$
\begin{align*}
\dot{x} & =\frac{R}{2}\left(V_{r}+V_{l}\right) \cos \theta,  \tag{1a}\\
\dot{y} & =\frac{R}{2}\left(V_{r}+V_{l}\right) \sin \theta,  \tag{1b}\\
\dot{\theta} & =\frac{R}{2}\left(V_{r}-V_{l}\right), \tag{1c}
\end{align*}
$$

This is the differential drive robot model. Here, $x, y$ and $\theta$ is the process of x position, y position of the robot change and the process of robot turning respectively. So, this is a model in terms of mapping control on to states, "Based on the limitations of Eqs. (1a), (1b) and (1c) (Lavalle, 2006), unicycle model has been used", which successfully overcome the
wheel velocities issue. Then, translational velocity (v), which is speed and angular velocity $(\omega)$ has been considered instead of considering wheel velocities. So, v and $\omega$ are two new input. Now, to map them a second order unicycle dynamics is needed, which are

$$
\begin{align*}
& \dot{x}=v \cos \theta  \tag{2a}\\
& \dot{y}=v \sin \theta  \tag{2b}\\
& \dot{\theta}=\omega \tag{2c}
\end{align*}
$$

But, "Equation (2a), (2b) and (2c) are not the differential drive model", $v$ and $\omega$ of equation (2) is the control input but $\mathrm{V}_{\mathrm{r}}$ and $\mathrm{V}_{1}$ of equation (1a) are actual control parameters. "Both the control parameter have been mapped to obtain the following equations

$$
\begin{align*}
& v=\frac{R}{2}\left(v_{r}+v_{l}\right)  \tag{3a}\\
& \frac{2 v}{R}=v_{r}+v_{l}  \tag{3b}\\
& \omega=\frac{R}{L}\left(v_{r}-v_{l}\right)  \tag{4a}\\
& \frac{\omega L}{R}=v_{r}-v_{l} \tag{4b}
\end{align*}
$$

Here, equation (3b) connects the translational velocity to the real velocities. Equation (3b) and (4b) are linear equations". This has been solved to find the desired model.

$$
\begin{align*}
& v_{r}=\frac{2 v+\omega L}{2 R}  \tag{5}\\
& v_{l}=\frac{2 v-\omega L}{2 R} \tag{6}
\end{align*}
$$

So, equation (5) and equation (6) is the desired robot model, where $v$ and $\omega$ are designed parameters, L and R is known measured parameters for the robot. And both kinds of parameters have been mapped onto the actual inputs.

### 2.2 Odometry

Odometry is the most commonly used navigation system to find the position of mobile robot. After getting the model most important issue is now to know the position of the robot and odometry is the key to grab this point. The main advantages are - short time accuracy, cheap and high sampling rates (Borenstein et al., 1996). So, odometry is a measuring technique of wheel rotation as a function of time, which means to keep track of robot's position and orientation (Ravikumar et. al, 2013). Now, sensors have been used to do this. There are different types of internal and external sensors. GPS is very suitable external sensor, which gives the information of position. But, robot is using for indoor facilities. So, there are no GPS signals. Moreover, in some cases GPS in not enough to give the position and orientation to any high level of loyalty. So, need a typically coupled external sensor with internal sensors. And then, wheel encoder has been used to find the odometry.

### 2.3 Wheel Encoder

Wheel encoder gives the distance moved by each wheel. The helpmate's position has been pointed out by the number of revolutions the wheels are doing in a certain amount of time. Now, "The center has turned by the robot is denoted by $D_{c}$, expressed in the following equation

$$
\begin{equation*}
D_{c}=\frac{\left(D_{l}+D_{r}\right)}{2} \tag{7}
\end{equation*}
$$

Where, $D_{1}$ is the distance of left wheel has turned and $D_{r}$ is the distance of right wheel has turned". And it has been considered that wheels turning at a constant rate and driving at a constant velocity. Means v and $\omega$ are constant and each wheel has N pulse per revolution (Egerstedt, 2014).
Now to find $D_{1}$ and $D_{r}$, difference of pulse has been calculated, which is shown in equation (8a) and (8b). For left wheel

$$
\begin{equation*}
\Delta p u l s e_{l}=\text { pulse }_{l_{\text {new }}}-p_{\text {old }} \tag{8a}
\end{equation*}
$$

For right wheel

$$
\begin{equation*}
\Delta p u l s e_{r}=\text { pulse }_{r_{\text {new }}}-p_{r_{\text {old }}} \tag{8b}
\end{equation*}
$$

Based on equation (8a) and (8b) distance can be measured for wheels

$$
\begin{align*}
& D_{l}=2 \Pi R \frac{\Delta p u l s e}{N}  \tag{9a}\\
& D_{r}=2 \Pi R \frac{\Delta p u l s e}{N} \tag{9b}
\end{align*}
$$

So, this actually gives as a way of mapping pulse on to distances traveled. Now to map this distance into new position and orientation following equations are required which made the relation between robot model and wheel encoders.

$$
\begin{align*}
x_{\text {new }}^{\prime} & =x+D_{c} \cos (\theta)  \tag{10a}\\
y_{\text {new }}^{\prime} & =y+D_{c} \sin (\theta)  \tag{10b}\\
\theta_{\text {new }}^{\prime} & =\theta+\frac{D_{r}-D_{l}}{L} \tag{10c}
\end{align*}
$$

Using the value of $x, y, \theta, D_{c}, D_{r}, D_{1}$ the new values have been calculated, which are the new position and new orientation of the robot.

### 2.4 Sensors

After getting the position of the robot, it is need to know the world around the robot. An infrared proximity sensor is dealing with this problem. Now, IR sensor has a definite range, within that range it can detect all the obstacles. Sensors measure distances in different direction. In Fig. 1 there are two obstacles represented by i and ii. $d_{1}, d_{2}$ are the distances of the obstacles and $\theta_{1}, \theta_{2}$ are the angles to the obstacle-i and obstacle-ii respectively according to the robot coordinate system.

Now, in robot coordinate system angles are relative to the robot's heading. In Fig. 1 blue line is the X -axis for the robot then $\theta_{2}$ is measuring relative to robot's X -axis and robot is heading left. So, to know the position of the obstacles globally this condition should be in consideration.

Position of the helpmate $\mathrm{x}, \mathrm{y}$ and $\theta$ already has been calculated. Actual direction of obstacle-ii is $\left(\theta_{2}+\theta\right)$. So, "Global position of obstacle-ii calculated by

$$
\begin{align*}
& x_{2}=x+d_{2} \cos \left(\theta_{2}+\theta\right)  \tag{11}\\
& y_{2}=y+d_{2} \sin \left(\theta_{2}+\theta\right) \tag{12}
\end{align*}
$$

In equation (11) and (12) $x_{2}$ and $y_{2}$ is the global position of obstacle-ii, $x$ and $y$ is the position of the robot, $d_{2}$ is the distance to the obstacle, $\theta_{2}$ is the angle relative to the robots heading, $\theta$ is the robots main orientation" (Egerstedt, 2014).


Fig. 1. Obstacles of the Helpmate within IR sensor range.

## 3 COORDINATE FRAME

Coordinate frames are required to find the location and orientation of the robot. It can be classified in many ways.

### 3.1 World Coordinate frame

This is a global frame which is centered at the origin $(0,0)$. The location and orientation of the robot is given by $x, y$ and $\theta$ respectively with respect to the world frame. As shown Fig. 2 (a) the helpmate orientation $\theta$ is given with respect to X -axis of the world frame.


Fig. 2. (a) Location and Orientation of Helpmate Robot with respect to World Frame, (b) Position and Orientation of the five sensors with respect to the Robot frame.

### 3.2 Robot Coordinate frame

The coordinate frame is located right at the robot called local frame or robot coordinate frame. In the Fig. 2(b) shows the helpmate's location and orientation with respect to the robot frame with their sensors position.

### 3.3 Sensor Coordinate frame

There are five sensors and each sensor has its own coordinate frame. The location of the sensor is the origin of this coordinate frame and X -axis of that frame lies with the orientation of the sensors. In Fig. 3(a) sensor i's frame (e.g. i=1, 2, 3, 4, 5) the robot measures the distance, $\mathrm{d}_{\mathrm{i}}$ in X direction and 0 in the Y direction.


Fig. 3. (a) Sensor i's Frame, (b) Example of rotation and translation in a coordinate frame.

Now to find the trajectory of the robot an informed decision has been made by controller. After obtaining the points i in the sensor's coordinate, the obstacle have been identified at that location in the world. Then robot can make the decision of movement.

The important fact that required clarifying is the transformation between different coordinate frames. "Two coordinate frames have been related by the following equation:

$$
\begin{equation*}
X_{C 1}=R(\theta) X_{C 2} \tag{13}
\end{equation*}
$$

Where, $\mathrm{X}_{\mathrm{C} 1}=\left[\begin{array}{l}x \\ y \\ \theta\end{array}\right], \mathrm{X}_{\mathrm{C} 2}=\left[\begin{array}{l}x \\ y \\ \theta\end{array}\right]$, and $\mathrm{R}(\theta)=\left[\begin{array}{ccc}\cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1\end{array}\right]$
Here, $\mathrm{X}_{\mathrm{C} 1}$ and $\mathrm{X}_{\mathrm{C} 2}$ represent the position and orientation of any point of first and second coordinate frame respectively. And $\mathrm{R}(\theta)$ is a rotation matrix (Dhaouadi, and Hatab, 2013)". But, the work has been accomplished by rotation and transformation in 2D. In the Fig. 3(b) (x,y) pretended as a vector. The vector has been multiplied by a transformation matrix, then translated by $x^{\prime}$ and $y^{\prime}$ and then rotated by $\theta^{\prime}$.

$$
\mathrm{R}\left(\mathrm{x}^{\prime}, \mathrm{y}^{\prime}, \theta^{\prime}\right)=\left[\begin{array}{ccc}
\cos \theta & -\sin \theta & \mathrm{x}^{\prime}  \tag{14}\\
\sin \theta & \cos \theta & y^{\prime} \\
0 & 0 & 1
\end{array}\right]
$$

And this transformation matrix is needed in order to transform from sensors reference frame to the world reference frame. The entire calculation has been calculated by the following equation (De la Croix, 2014).

$$
\left[\begin{array}{c}
X_{d i}  \tag{15}\\
Y_{d i} \\
1
\end{array}\right]=\mathrm{R}(\mathrm{x}, \mathrm{y}, \theta) \mathrm{R}\left(\mathrm{x}_{\mathrm{si}}, \mathrm{y}_{\mathrm{si}}, \theta_{\mathrm{si}}\right)\left[\begin{array}{c}
d_{i} \\
0 \\
1
\end{array}\right]
$$

Now after computing vectors from the robot to each one of five points; all vectors have been summed up. And the resulting vector pointing away from the obstacle. Then orientation of the vector and PID controller has been used to steer the Helpmate in the direction of the free space.

## 4 NAVIGATION

For a complete navigation system the four major controllers has been used to take the robot into goal location, such as: go-to-goal controller, avoid obstacles controller, blended controller, and follow obstacles controllers. Equation (2a), (2b), (2c) is a unicycle model of differential drive robot, where linear velocity is constant. "Heading has been controlled by equation (2c) and the error is defined by

$$
\begin{equation*}
e=\theta_{d}-\theta \tag{16}
\end{equation*}
$$

Where desired angle is represented by

$$
\begin{equation*}
\theta_{d}=\arctan \frac{y_{g}-y}{x_{g}-x} \tag{17}
\end{equation*}
$$

In equation (17), x and y is robot's location, $\mathrm{x}_{\mathrm{g}}$ and $\mathrm{y}_{\mathrm{g}}$ is goal location". And "PID controller has been used which acting an error in

$$
\begin{equation*}
\omega=\operatorname{PID}(e) \tag{18}
\end{equation*}
$$

Which gives the update of the heading (Egerstedt, 2014)".
Now, without colliding the heading has been controlled by two arbitration mechanisms like hard switching and blending. Already the robot's position, obstacles and goal location has been obtained. The Heading of the robot just to goal or just going straight away from the obstacles is a hard switching (Egerstedt, 2014). Combining go-to-goal and avoid obstacle is called blending. "It can be represented by following equation

$$
\begin{equation*}
u_{a o, g t g}=\alpha u_{a o, n}+(1-\alpha) u_{g t g, n}, 0 \leq \alpha \leq 1 \tag{19}
\end{equation*}
$$

Equation (19) is the summing fraction of obstacle avoidance vector and go-to-goal vector. Where, $\mathrm{u}_{\mathrm{a} 0, \mathrm{n}}$ is the normalized version of the vector". And this equation also points both in the direction of the goal and in the direction away from the obstacle (De la Croix, 2014).

Now, by using go-to-goal, avoid obstacle and blended controller it is possible to go the desired location without colliding the obstacle but for concave or labyrinths these controllers are no longer exist. To overcome these follow wall controller has been used which successfully avoid the concave or labyrinths and reach its desired goal.

For this case, IR sensors estimate a section of obstacle. Then helpmate steers towards the resulting vector of tangential and perpendicular vector to the wall. After estimating and maintaining spacing from the wall the important issue is movement of the helpmate to clockwise and counter-clockwise direction. "And mathematically it can be represent as

$$
\begin{align*}
& u_{F W}^{c}=\alpha R\left(-\frac{\Pi}{2}\right) u_{A O}=\alpha\left[\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right] u_{A O},  \tag{20}\\
& u_{F W}^{c c}=\alpha R\left(\frac{\Pi}{2}\right) u_{A O}=\alpha\left[\begin{array}{cc}
0 & -1 \\
1 & 0
\end{array}\right] u_{A O}, \tag{21}
\end{align*}
$$

Here, c stands for clockwise, cc stands for counter clockwise, $\alpha$ is a scaling factor, $R(\theta)$ is a rotation matrix, AO stands for avoid obstacle".

But movement of clockwise and counter clockwise has been made by go-to-goal behavior and based on the behavior the angle has been calculated through the inner product.

$$
\begin{equation*}
\langle v, \omega\rangle=v^{T} \omega=\|v\|\|\omega\| \cos (\angle(v, \omega)) \tag{22}
\end{equation*}
$$

Where,

$$
\begin{aligned}
& \text { angle }<\pi / 2 \text {, cosine is positive } \\
& \text { angle }>\pi / 2 \text {, cosine is negative }
\end{aligned}
$$

Then

$$
\begin{align*}
& \left\langle u_{G T G}, u^{c} F W\right\rangle>0,  \tag{23}\\
& \left\langle u_{G T G}, u^{c c} F W\right\rangle>0, \tag{24}
\end{align*}
$$

Now based on the sign of the inner product equation (23) and (24) will be satisfied and dissatisfied (Egerstedt, 2014).

Then important point is robot has to be stopped at goal location. And for these two issues of the robots have been calculated. "Issues are progress of the robot and clear shot has been representing by the following equations respectively

$$
\begin{gather*}
\left\|x_{c}-x_{g}\right\|<\left\|x(\tau)-x_{g}\right\|  \tag{25}\\
\left\langle u_{A O}, u_{G T G}\right\rangle>0 \tag{26}
\end{gather*}
$$

Where, $\mathrm{x}_{\mathrm{c}}$ is the current position of the helpmate, $\mathrm{x}_{\mathrm{g}}$ is the goal location of the helpmate, $\tau$ is the time of last switch (Egerstedt, 2014)".

The FSM (Jones et al., 1998) and the trajectory of the helpmate robot are shown in Fig. 4 and Fig. 5 respectively.


Fig. 4. A Finite State Machine


Fig.5. Progress towards the goal (L) of helpmate robot, and (i),(ii),(iii),(iv) denotes different position of helpmate with different controller

Fig. 6 (a) describes the situation of position-i of Fig. 5. Helpmate first tries to go into the goal. This has been represented by $u_{\text {gtg }}$. There is also a avoid obstacle vector, $\mathrm{u}_{\mathrm{ao}}$. And resulting heading is $\mathrm{u}_{\mathrm{gtg} \text { _ao }}$ vector. In Fig. 6 (b) some progress has been made and current location of the helpmate is closer with respect to the position-i. And follow-wall controller has been used to go from position-ii to position-iii.

After using go-to-goal and avoid obstacle controller helpmate has been switched in follow-wall controller because of no progress behavior. Angle between $u_{\mathrm{gtg}}$ and $u_{\mathrm{ao}}$ is less than $\pi$. This has been shown in Fig. 6 (c). And using equation (23) and (24) helpmate steered from position-iii to position-iv.


Fig. 6. (a) Progression towards the goal, (b) Progression with respect to positioni, (c) Switching to follow wall, (d) New progress
$\mathrm{u}_{\mathrm{fw}, \mathrm{r}}$ and $\mathrm{u}_{\mathrm{fw}, \mathrm{l}}$ vector are no longer inside between the go-to-goal vector and avoid obstacle vector in Fig. 6 (d). So, robot is no longer in sliding mode. Helpmate switch back it's go-to-goal controller and reached its desired goal (De la Croix, 2014). The architecture of helpmate system is shown in Fig. 7.


Fig. 7. Helpmate Architecture

## 5 MECHANICAL AND ELECTRICAL DESIGN

The designed helpmate is a unique prototype model which accomplished its functionality successfully. The prototype is suitable for unstructured condition to take the medicines from one place to another by avoiding obstacles. Implemented helpmate is a differential drive robot with an attached tray. The tray is attached with the robot by three light stick. The plastic tray is not packed with the robot, so it is easy to maintenance. It is very available and also reducing the cost and weight with respect to the previous helpmates. That is as shown in Fig. 8.


Fig. 8. Plastic tray.
Chassis, wheels, motors are not permanently attached. So due to any mechanical fault it's easy to replace. Moreover, helpmate runs with Beaglebone Black, motors, sensors, wires, power supply and wifi. The helpmate with wifi module will easily run on 8AA 1900 mAh batteries. The wifi allows connecting it to the network and driving around wirelessly.

An H-bridge has been used to controls the motor speed and velocity after getting commands from the Beaglebone Black. Voltage regulator on the prototype brings the battery voltage down to 5 V . The Beaglebone Black with wifi draws around 1A current. The LD1085 has worked well to do this and need at least 6.5 volts to work properly. Because according to the output voltage LD1085 needs to be supplied at least 1.5 volts above. An additional 0.7 volt drop across the rectifier, in this case 6 NiMH AA batteries
works well instead of 8 batteries. But it has been studied that to get better torque and speed out of the motor around 9 V to 12 V works nicely with loaded condition. So some extra battery cells have been used. The implemented helpmate prototype is show in Fig. 9.


Fig. 9. (a) Side view, (b) Front view, (c) Wiring of lower chassis, (d) Wiring of upper chassis, (e)
Differential Drive prototype, (f) Helpmate prototype.

## 6 CONCLUSIONS

This paper presents a detail mathematical approach of a differential drive prototype helpmate robot. The benefit of this model is minimization of the complexity of the helpmate robot both in hardware and navigation system by a cost efficient way with respect to the previous models. At the same time the mathematical analysis has been developed based on some basic assumption regarding the functionality of the wheel. Applied navigation system is suitable for the indoor environment. Moreover, mathematically it has been shown that robot can reach the goal without colliding obstacles with necessary figures.

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## References

Arvin, F., Samsudin, K., Nasseri, M. A. (2009, July 25-26). Design of a Differential-Drive Wheeled Robot Controller with Pulse-Width Modulation. Conference on Innovative Technologies in Intelligent Systems and Industrial Applications, Monash University, Sunway campus, Malaysia. pp. 143-147.
Braunl, T. (2006). EMBEDDED ROBOTICS, Mobile Robot Design and Applications with Embedded Systems, $2^{\text {nd }}$ edition. Springer.
Balkcom. D.J. and Mason. M. T.(2002,March) Time optimal trajectories for bounded velocity differential drive vehicles. The International Journal of Robotics Research, 21(3):199-218.

Bishop, R.H. (2006). Mechatronics: An Introduction. ISBN 0-8493-6358-6, CRC Press.
Borenstein, J., Everett, H. R., Feng, L. \& Wehe, D.(1996, September). Mobile Robot Positioning: Sensors and Techniques. Journal of Robotic Systems. 14(4), 231-249.
Cho, G., and Ryeu, J. (2006, March). An Efficient Method to Find a Shortest Path for a Car-Like Robot. International Journal of Multimedia and Ublqultous Engineering. Vol. 1, No. 1.
De la Croix, J.-P. (2014). Programming \& Simulation lecture 2: Differential-Drive, Odometry, and IR. Control of Mobile Robots. Coursera.

De la Croix, J.-P. (2014). Programming \& Simulation lecture 4: Obstacle Avoideance. Control of Mobile Robots. Coursera.

De la Croix, J.-P. (2014). Programming \& Simulation lecture 5: Arbitration. Control of Mobile Robots. Coursera.

De la Croix, J.-P. (2014). Programming \& Simulation lecture 7: Navigation. Control of Mobile Robots. Coursera.

Dhaouadi, R. \& Hatab, A. A. (2013, September 19). Dynamic Modelling of DifferentialDrive Mobile Robots using Lagrange and Newton-Euler Methodologies: A Unified Framework. Advance in Robotics \& Automation. Volume 2, Issue 2. ISSN 21689695.

Egerstedt, M. (2014). Module 2: Mobile Robots, Control of Mobile Robots. Coursera.

Egerstedt, M. (2014). Module 6: The Navigation Problem. Control of Mobile Robots. Coursera.

Fox, D., Burgard, W. \& Thrun, S. (1996). Controlling Synchro-drive Robots with the Dynamic Window Approach to Collision Avoidance. IEEE/RSJ International Conference on Intelligent Robots and Systems. pp.1280-1287.

Jones, J.L., Seiger, B.A. \& Flynn, A.M. (1998, November). Mobile Robots: Inspiration to Implementation. ISBN 10-1568810970, Natikc, Massachusetts : A K Peters.

Lavalle, S. M. (2006). Planning Algorithms. Cambridge University Press.
Ravikumar, T. M., Saravanan, R. \& Nirmal, N. (2013, December). Modeling and Optimization of Odometry Error in a Two Wheeled Differential Drive Robot. International Journal of Scientific and Research Publication. Volume 3, Issue 12. ISSN 2250-3153.

Song, J.B., and Byun, K.S. (2006, December). Design and Control of an Omnidirectional Mobile Robot with Steerable Omnidirectional Wheels. Mobile Robots, Moving Intelligence. ISBN: 3-86611-284-X, pp. 576.
Siegwart, R. \& Nourbakhsh, I. R. (2004). Introduction to Autonomous Mobile Robots. England : The MIT Press.

Xu, Y., Ou, Y. (2005). Control of Single Wheel Robot. New York: Springer, ISBN- 13 978-3-540-28184-9.

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