On some results on fuzzy δ-connected space in fuzzy topological space on fuzzy sets

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Abstract :

The aim of this paper to introduce and study fuzzy δ-open set and the relations of some other class of fuzzy open sets , introduce and study fuzzy δ-connected space , and the relations of some other class of fuzzy connected spaces of types (R-connected , θ-connected , γ-connected , Δ-connected) on fuzzy topology space on fuzzy set and study some properties and theorems on this subject

Introduction :

The concept of fuzzy set was introduced by Zedeh in his classical paper in 1965. The fuzzy topological space was introduced by Chang in 1968. Zahran has introduced the concepts of fuzzy δ-open sets, fuzzy δ-closed sets, fuzzy regular open sets, and fuzzy regular closed sets. And Luay A.Al.Swidi, Amed S.A.Oon introduced the notion of γ-open set, fuzzy γ-closed set and studied some of its properties. M. N. Mukherjee and S. P. Sinha introduced the concept of fuzzy θ-open set, fuzzy θ-closed set.And Shyamal Debnath introduced the concepts of fuzzy δ-semi connected space. The purpose of the present paper is to introduce and study the concepts of fuzzy δ-open sets and some types of fuzzy open set and relationships between of them and study of fuzzy δ-connected space and some types of fuzzy connected space and relationships between of them and we examine the validity of the standard results.

1. Fuzzy topological space on fuzzy set

1.1 Definition (Zadeh1965)

Let X be a non empty set, a fuzzy set $\tilde{A}$ in X is characterized by a function $\mu_{\tilde{A}} : X \to I$, where $I = [0,1]$ which is written as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X , 0 \leq \mu_{\tilde{A}}(x) \leq 1\}$, the collection of all fuzzy sets in X will be denoted by I^X,that is $I^X = \{ \tilde{A} : \tilde{A}$ is a fuzzy sets in X$\}$ where $\mu_{\tilde{A}}$ is called the membership function .

1.2 Proposition (Wong,C. K, 1974)

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets in X with membership functions $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ respectively then for all $x \in X$: -

1. $\tilde{A} \subseteq \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$.
2. $\tilde{A} = \tilde{B} \leftrightarrow \mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x)$.
3. $\tilde{C} = \tilde{A} \cap \tilde{B} \leftrightarrow C(x) = \min\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$.
4. $\tilde{D} = \tilde{A} \cup \tilde{B} \leftrightarrow D(x) = \max\{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$.
5. $\tilde{B}^c$ the complement of $\tilde{B}$ with membership function $\mu_{\tilde{B}^c}(x) = \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)$.

1.3 Definition (Kandil1,2012)

A fuzzy point x_r is a fuzzy set such that :

$\mu_{x_r}(y) = r > 0$ if $x = y$, $\forall\, y \in X$ and $\mu_{x_r}(y) = 0$ if $x \neq y$, $\forall\, y \in X$

The family of all fuzzy points of $\tilde{A}$ will be denoted by $FP(\tilde{A})$.

1.4 Remark : (Chakraborty M. K ,1992)

Let $\tilde{A} \in I^X$ then $P(\tilde{A}) = \{ \tilde{B} : \tilde{B} \in I^X, \mu_{\tilde{B}}(x) \leq \mu_{\tilde{A}}(x) \} \forall x \in X$.

1.5 Definition (Chakraborty M. K ,1992)

A collection $\tilde{T}$ of a fuzzy subsets of $\tilde{A}$, such that $\tilde{T} \subseteq P(\tilde{A})$ is said to be fuzzy topology on $\tilde{A}$ if it satisfied the following conditions

1. $\tilde{A}$, $\tilde{\Phi} \in \tilde{T}$
2. If $\tilde{B}, \tilde{C} \in \tilde{T}$ then $\tilde{B} \cap \tilde{C} \in \tilde{T}$
3. If $\tilde{B}_j \in \tilde{T}$ then $\cup_j \tilde{B}_j \in \tilde{T}$, $j \in J$

$(\tilde{A}, \tilde{T})$ is said to be Fuzzy topological space and every member of $\tilde{T}$ is called fuzzy open set in $\tilde{A}$ and its complement is a fuzzy closed set .

1.6 Definition (Chaudhuri, 1993)

Let $\tilde{B} \in P(\tilde{A})$ ($x_r \in FP(\tilde{A})$) , then $\tilde{B}$ [x_r] is said to be :

- Maximal fuzzy point in $\tilde{A}$ if for each $x \in X$, $\mu_{x_r}(x) \neq 0$ then $\mu_{x_r}(x) = \mu_{\tilde{A}}(x)$.
- Maximal fuzzy set in $\tilde{A}$ if for each $x \in X$, $\mu_{\tilde{B}}(x) \neq 0$ then $\mu_{\tilde{B}}(x) = \mu_{\tilde{A}}(x)$

2. On fuzzy δ-open set

2.1 Definition

A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be

- Fuzzy δ-open set if $\mu_{Int(Cl(\tilde{B}))}(x) \leq \mu_{\tilde{B}}(x)$
- Fuzzy δ-closed set if $\mu_{\tilde{B}}(x) \leq \mu_{Cl(Int(\tilde{B}))}(x)$

2.2 Remark

The family of all fuzzy δ-open sets [resp. fuzzy δ-closed sets] in a fuzzy topological space $(\tilde{A}, \tilde{T})$ will be denoted by $F\delta O(\tilde{A})$ [resp. $F\delta C(\tilde{A})$]

2.3 proposition

Every fuzzy open set [fuzzy closed set] is fuzzy δ-open set [fuzzy δ-closed set] .

Proof : Obvious

2.4 Remark

The converse of proposition(2.3) is not true in general as the following example shows

2.5 Example

Let $X = \{a, b, c\}$ and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$, $\tilde{E}$, $\tilde{F}$ be fuzzy subsets of $\tilde{A}$ where:

$\tilde{A} = \{(a, 0.8), (b, 0.8), (c, 0.8)\}$, $\tilde{B} = \{(a, 0.1), (b, 0.1), (c, 0.2)\}$, $\tilde{C} = \{(a, 0.2), (b, 0.1), (c, 0.2)\}$ $\tilde{D} = \{(a, 0.3), (b, 0.3), (c, 0.2)\}$, $\tilde{E} = \{(a, 0.4), (b, 0.4), (c, 0.5)\}$, $\tilde{F} = \{(a, 0.3), (b, 0.3), (c, 0.3)\}$

The fuzzy topologies defined on $\tilde{A}$ are $\tilde{T} = \{\tilde{\phi}, \tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}\}$,The fuzzy set $(\tilde{E})$ $[\tilde{F}]$ in fuzzy topological space $(\tilde{A}, \tilde{T})$ is fuzzy δ-open set [fuzzy δ-closed set] but not fuzzy open set [fuzzy closed set] .

2.6 Definition (Seok and Sang , 2012)

A fuzzy set $\tilde{B}$ in a fuzzy topological space $(\tilde{A}, \tilde{T})$ is called fuzzy δ-neighborhood (δnbhd) of a fuzzy point x_r in $\tilde{A}$ if there is a fuzzy δ-open set $\tilde{G}$ in $\tilde{A}$ such that $\mu_{x_r}(x) \leq \mu_G(x) \leq \mu_{\tilde{B}}(x)$, $\forall\, x \in X$.

2.7 Proposition

Every fuzzy neighborhood $\tilde{B}$ of x_r is fuzzy δ-neighborhood.

Proof : Obvious

2.8 Definition (Seok and Sang , 2012)

Let $\tilde{B}$ be a fuzzy set in a fuzzy topological space $(\tilde{A}, \tilde{T})$ then:

- The δ – closure of $\tilde{B}$ is denoted by $(\delta cl(\tilde{B}))$ and defined by
  $\mu_{\delta cl(\tilde{B})}(x) = \min\{\mu_{\tilde{F}}(x) : \tilde{F}$ is a fuzzy δ – closed set in $\tilde{A}$, $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{F}}(x)\}$.
- The δ – interior of $\tilde{B}$ is denoted by $(\delta int(\tilde{B}))$ and defined by
  $\mu_{\delta int(\tilde{B})}(x) = \max\{\mu_G(x) : \tilde{G}$ is a fuzzy δ – open set in $\tilde{A}$, $\mu_G(x) \leq \mu_{\tilde{B}}(x)\}$.

2.9 Theorem (Seok and Sang , 2012)

Let $\tilde{B}$, $\tilde{C}$ are fuzzy sets in a fuzzy topological space $(\tilde{A}, \tilde{T})$ then ;

1. $\mu_{\delta cl(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{\delta cl(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$.
2. If $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}}(x)$ then $\mu_{\delta cl(\tilde{B})}(x) \leq \mu_{\delta cl(C)}(x)$.
3. $\mu_{\tilde{B}}(x) \leq \mu_{\delta cl(\tilde{B})}(x)$.
4. $\mu_{\delta cl(\delta cl(\tilde{B}))}(x) = \mu_{\delta cl(\tilde{B})}(x)$.
5. $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x) \leq \min\{\mu_{\delta cl(\tilde{B})}(x), \mu_{\delta cl(C)}(x)\}$.
6. $\mu_{\delta cl(max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x)\})}(x) = \max\{\mu_{\delta cl(\tilde{B})}(x), \mu_{\delta cl(C)}(x)\}$.

2.10 Theorem (Seok and Sang, 2012)

Let $\tilde{B}$, $\tilde{C}$ are fuzzy sets in a fuzzy topological space $(\tilde{A}, \tilde{T})$ then ;

1. $\mu_{\delta int(\tilde{\emptyset})}(x) = \mu_{\tilde{\emptyset}}(x)$ and $\mu_{\delta int(\tilde{A})}(x) = \mu_{\tilde{A}}(x)$.
2. If $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{C}}(x)$ then $\mu_{\delta int(\tilde{B})}(x) \leq \mu_{\delta int(C)}(x)$.
3. $\mu_{\delta int(\tilde{B})}(x) \leq \mu_{\tilde{B}}(x)$.

4. $\mu_{\delta int(\delta int(\tilde{B}))}(x) = \mu_{\delta int(\tilde{B})}(x)$.
5. $\mu_{\delta int(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x) = \min\{\ \mu_{\delta int(\tilde{B})}(x)\ ,\ \mu_{\delta int(C)}(x)\ \}$.
6. $\max\{\ \mu_{\delta int(\tilde{B})}(x)\ ,\ \mu_{\delta int(C)}(x)\ \} \leq \mu_{\delta int(max\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{C}}(x))}(x)$.

2.11 Definition (K.K.Azad ,1981)

A fuzzy set B̃ of a fuzzy topological space (Ã ,T̃) is said to be :-

1) Fuzzy regular open [Fuzzy regular closed] set if :

$\mu_{\tilde{B}}(x) = \mu_{Int(Cl(\tilde{B}))}(x)$ $[\mu_{\tilde{B}}(x) = \mu_{Cl(Int(\tilde{B}))}(x)]$, The family of all fuzzy regular open [fuzzy regular closed]set in Ã will be denoted by FRO(Ã)[FRC(Ã)]. (K.K.Azad ,1981)

2) Fuzzy Δ-open set if for every point $x_r \in \tilde{B}$ there exist a fuzzy regular semi-open set $\tilde{U}$ in Ã such that $\mu_{x_r}(x) \leq \mu_{\tilde{U}}(x) \leq \mu_{\tilde{B}}(x)$, B̃ is called [Fuzzy Δ-closed] set if its complement is Fuzzy Δ-open set the family of all Fuzzy Δ-open [Fuzzy Δ-closed] sets in Ã will be denoted by FΔO(Ã)[FΔC(Ã)]. (Shahla H. K., 2004)

3) Fuzzy $\gamma - open$ [$\gamma - closed$] set if

$\mu_{\tilde{B}}(x) \leq \max\{\ \mu_{Int(Cl(\tilde{B}))}(x)\ ,\ \mu_{Cl(int(\tilde{B}))}(x)\}$, $[\mu_{\tilde{B}}(x) \geq \min\{\mu_{Int(Cl(\tilde{B}))}(x)\ ,\ \mu_{Cl(int(\tilde{B}))}(x)\}]$, The family of all fuzzy γ – open [fuzzy γ – closed] sets in Ã will be denoted by FγO(Ã) [FγC(Ã)]. (Luay A.Al.Swidi,Amed S.A.Oon,2011)

4) Fuzzy θ-open [θ-closed] set if $\mu_{\tilde{B}}(x) = \mu_{\theta Int(\tilde{B})}(x)$, $[\ \mu_{\tilde{B}}(x) = \mu_{\theta Cl(\tilde{B})}(x)]$, $\forall\ x \in X$

The family of all fuzzy θ-open (fuzzy θ-closed) sets in Ã will be denoted by FθO(Ã) [FθC(Ã)].

(M. N. Mukherjee and S. P. Sinha ,1991)

2.12 Proposition

Let (Ã , T̃) be a fuzzy topological space then :

1) Every fuzzy δ-open set (resp. fuzzy δ-closed set) is fuzzy Δ-open set (resp.fuzzy Δ-closed set) **[**fuzzy γ-open set (resp.fuzzy γ-closed set) **]**.
2) Every fuzzy θ-open set (resp. fuzzy θ-closed set) is fuzzy γ-open set (resp. fuzzy γ-closed set) **[**fuzzy δ-open set (resp. fuzzy δ-closed set , fuzzy Δ-open set (resp.fuzzy Δ-closed set) **]**
3) Every fuzzy regular open set (fuzzy regular closed set) is fuzzy δ-open set (resp. fuzzy δ-closed set) **[**fuzzy γ-open set(resp. fuzzy γ-closed set) , fuzzy Δ-open set(resp.fuzzy Δ-closed set)**]**

Proof **:** Obvious .

2.13 Remark

The converse of proposition (2.12) is not true in general as following examples shows

2.14 Examples

1) Let X = { a , b } and $\tilde{B}$, $\tilde{C}$, $\tilde{D}$ are fuzzy subset in $\tilde{A}$ where

$\widetilde{A}$ = { (a , 0.9) , (b , 0.9) } , $\widetilde{B}$ = { (a , 0.0) , (b , 0.7) } , $\widetilde{C}$ = { (a , 0.8) , (b , 0.0) }, $\widetilde{D}$ = { (a , 0.8) , (b , 0.7) , The fuzzy topology defined on $\widetilde{A}$ is $\tilde{T}$ = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$ }

- The fuzzy set $\widetilde{D}$ is a fuzzy Δ-open set but not fuzzy δ – open set (fuzzy regular open set , fuzzy θ – open set).
- let X = { a , b , c } and $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\widetilde{E}$ are fuzzy subset in $\widetilde{A}$ where
  $\widetilde{A}$ = { (a , 0.9) , (b , 0.9) , (c , 0.9) } , $\widetilde{B}$ = { (a , 0.3) , (b , 0.3) , (c , 0.4) }
  $\widetilde{C}$ = { (a , 0.4) , (b , 0.3) , (c , 0.4) } , $\widetilde{D}$ = { (a , 0.5) , (b , 0.5) , (c , 0.4) }
  $\widetilde{E}$ = { (a , 0.6) , (b , 0.6) , (c , 0.7), The fuzzy topology defined on $\widetilde{A}$ is
  $\tilde{T}$ = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\widetilde{E}$ }
- The fuzzy set $\widetilde{B}$ is a fuzzy γ – open set but not fuzzy δ – open set (fuzzy regular open set , fuzzy θ – open set).

2) The fuzzy set $\widetilde{E}$ in the example (2-5) is a fuzzy δ-open set but not fuzzy regular open set (fuzzy θ-open set).

3. fuzzy δ-separated set

3.1 Definition

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space and $\tilde{B}$, $\tilde{C}$ are fuzzy set in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are said to be fuzzy δ-separated sets if and only if min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$.

3.2 Theorem

If ($\tilde{A}$,$\tilde{T}$) is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ and $\widetilde{D}$ is a fuzzy set in $\tilde{A}$ then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\widetilde{D}}(x)$} and min{ $\mu_{\widetilde{D}}(x)$, $\mu_{\tilde{C}}(x)$} are fuzzy δ-separated sets in $\tilde{A}$.

Proof :

Since $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ **,** Then min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$,To prove min { $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\widetilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\widetilde{D}}(x))}(x)$ } = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(min\{\mu_{\tilde{C}}(x),\ \mu_{\widetilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{B}}(x),\ \mu_{\widetilde{D}}(x))}(x)$ } = $\mu_{\tilde{\emptyset}}(x)$

Since min{ $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\widetilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\widetilde{D}}(x))}(x)$ } $\leq$
min{min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\delta cl(\widetilde{D})}(x)$ }, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\widetilde{D}}(x))}(x)$ } =

min{min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$ }, $\mu_{(min\{\mu_{\delta cl(\widetilde{D})}(x),\ \mu_{\widetilde{D}}(x))}(x)$ } =

min{ $\mu_{\tilde{\emptyset}}(x)$, $\mu_{(min\{\mu_{\delta cl(\widetilde{D})}(x),\ \mu_{\widetilde{D}}(x))}(x)$ }= $\mu_{\tilde{\emptyset}}(x)$, then min{ $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$ } $\leq \mu_{\tilde{\emptyset}}(x)$

Hence min{ $\mu_{\delta cl(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$ }$=\mu_{\tilde{\emptyset}}(x)$

Similary min { $\mu_{\delta cl(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{D}}(x))}(x)$, $\mu_{(min\{\mu_{\tilde{B}}(x),\ \mu_{\tilde{D}}(x))}(x)$ } $= \mu_{\tilde{\emptyset}}(x)$

Then min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{D}}(x)$} and min{ $\mu_{\tilde{D}}(x)$, $\mu_{\tilde{C}}(x)$}are fuzzy δ-separated sets in Ã ■

3.3 Theorem

If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy δ-separated sets in Ã , M̃and Ñ are fuzzy set in Ã such that $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ and $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$ then M̃ and Ñ are fuzzy δ-separated sets in Ã .

Proof :

Since B̃ and C̃ are fuzzy δ-separated sets in Ã then

min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\tilde{M}}(x) \leq \mu_{\tilde{B}}(x)$ then $\mu_{\delta cl(\tilde{M})}(x) \leq \mu_{\delta cl(\tilde{B})}(x)$ then min { $\mu_{\delta cl(\tilde{M})}(x)$, $\mu_{\tilde{C}}(x)$} $\leq$ min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$, Hence min { $\mu_{\delta cl(\tilde{M})}(x)$, $\mu_{\tilde{C}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$

min { $\mu_{\delta cl(\tilde{M})}(x)$,$\mu_{(min\{\mu_{\tilde{C}}(x),\ \mu_{\tilde{N}}(x))}(x)$ } = min { min{ $\mu_{\delta cl(\tilde{M})}(x)$, $\mu_{\tilde{C}}(x)$ }, $\mu_{\tilde{N}}(x)$ } = min { $\mu_{\tilde{\emptyset}}(x)$, $\mu_{\tilde{N}}(x)$ } $= \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\tilde{N}}(x) \leq \mu_{\tilde{C}}(x)$ then min { $\mu_{\tilde{N}}(x)$, $\mu_{\tilde{C}}(x)$ }$= \mu_{\tilde{N}}(x)$

implies that min { $\mu_{\delta cl(\tilde{M})}(x)$, $\mu_{\tilde{N}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$, Similarly min { $\mu_{\delta cl(\tilde{N})}(x)$, $\mu_{\tilde{M}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$

Hence M̃ and Ñ are fuzzy δ-separated sets in Ã ■

3.4 Theorem

If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy sets in Ã then B̃ and C̃ are fuzzy δ-separated sets in Ã if and only if there exist fuzzy δ-closed sets Ẽ and F̃ in Ã such that $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{F}}(x)$}$= \mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\tilde{C}}(x)$, $\mu_{\tilde{E}}(x)$}$= \mu_{\tilde{\emptyset}}(x)$.

Proof

⇒ Suppose that B̃ and C̃ are fuzzy δ-separated sets in Ã

Implies that min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} $= \mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\tilde{B}}(x) \leq \mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\delta cl(\tilde{C})}(x)$,Then $\mu_{\tilde{E}}(x) = \mu_{\delta cl(\tilde{B})}(x)$ and $\mu_{\tilde{F}}(x) = \mu_{\delta cl(\tilde{C})}(x)$

Hence $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$, $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{F}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\tilde{C}}(x)$, $\mu_{\tilde{E}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

$\Leftarrow$ Since $\mu_{\tilde{B}}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\tilde{C}}(x) \leq \mu_{\tilde{F}}(x)$, Then $\mu_{\delta cl(\tilde{B})}(x) \leq \mu_{\tilde{E}}(x)$ and $\mu_{\delta cl(\tilde{C})}(x) \leq \mu_{\tilde{F}}(x)$

Implies that min {$\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}$\leq$ min {$\mu_{\tilde{E}}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

And min {$\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} $\leq$ min { $\mu_{\tilde{F}}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence min { $\mu_{\delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ therefore B̃ and C̃ are fuzzy δ-separated sets in Ã ■

3.5 Theorem

If (Ã ,T̃) is a fuzzy topological space, B̃ and C̃ are fuzzy sets in Ã then B̃ and C̃ are fuzzy δ-separated sets in Ã if B̃ and C̃ fuzzy δ-closed sets in Ã and min{ $\mu_{\tilde{B}}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$.

***<u>Proof</u>* :** Obvious

3.6 Remark : The converse of theorem (3.5) is not true in general .

3.7 Example

Let X = { a , b } and B̃ , C̃ , D̃ , Ẽ be fuzzy subsets of Ã where:

Ã = {(a ,0.2) ,(b ,0.5) } , B̃ = {(a ,0.0) , (b ,0.5) } ,C̃ = {(a ,0.2) , (b , 0.0) } ,

D̃ = {(a ,0.1) , (b ,0.0) } ,Ẽ = {(a ,0.0) , (b ,0.3) } The fuzzy topologies defined on Ã are

T̃ = { $\tilde{\Phi}$, Ã , B̃ , C̃ } ,Then D̃ and Ẽ are fuzzy δ-separated sets in Ã but not fuzzy δ-closed sets in Ã

3.8 Definition

1. If (Ã ,T̃) is a fuzzy topological space and B̃ , C̃ are fuzzy set in Ã then B̃ and C̃ are said to be fuzzy R-separated sets if and only if
   min { $\mu_{Rcl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{Rcl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$.

2. If (Ã ,T̃) is a fuzzy topological space and B̃ , C̃ are fuzzy set in Ã then B̃ and C̃ are said to be fuzzy Δ-separated sets if and only if
   min { $\mu_{\Delta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\Delta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$.
3. If (Ã ,T̃) is a fuzzy topological space and B̃ , C̃ are fuzzy set in Ã then B̃ and C̃ are said to be fuzzy γ-separated sets if and only if
   min { $\mu_{\gamma cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\gamma cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$.

4. If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space and $\tilde{B}$, $\tilde{C}$ are fuzzy set in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are said to be fuzzy θ-separated sets if and only if
   min { $\mu_{\theta cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$ and min { $\mu_{\theta cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$} = $\mu_{\tilde{\emptyset}}(x)$.

3.9 Theorem

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space, $\tilde{B}$ and $\tilde{C}$ are fuzzy R-separated sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets.

Proof :

Suppose that $\tilde{B}$ and $\tilde{C}$ are fuzzy R-separated sets in $\tilde{A}$

Then min{ $\mu_{\boldsymbol{R}cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\boldsymbol{R}cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Since $\mu_{\boldsymbol{R}cl(\tilde{B})}(x)$ and $\mu_{\boldsymbol{R}cl(\tilde{C})}(x)$ are fuzzy R-closed sets in $\tilde{A}$

Implies that by proposition (2.12) min{ $\mu_{\boldsymbol{\delta}cl(\tilde{B})}(x)$, $\mu_{\tilde{C}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\boldsymbol{\delta}cl(\tilde{C})}(x)$, $\mu_{\tilde{B}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$

Hence, $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated in $\tilde{A}$. ■

3.10 Remark

The converse of theorem (3.9) is not true in general as following examples shows

3.11 Example

Let X = { a , b , c } and $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\widetilde{E}$, $\widetilde{F}$ are fuzzy subset in $\widetilde{A}$ where
$\widetilde{A}$ = { (a , 0.8) , (b , 0.8) , (c , 0.8) }, $\widetilde{B}$ = { (a , 0.5) , (b , 0.0) , (c , 0.5) }
$\widetilde{C}$ = { (a , 0.0) , (b , 0.2) , (c , 0.0) } , $\widetilde{D}$ = { (a , 0.5) , (b , 0.2) , (c , 0.5) }
$\widetilde{E}$ = { (a , 0.4) , (b , 0.0) , (c , 0.4) }, $\widetilde{F}$ = { (a , 0.0) , (b , 0.1) , (c , 0.0) }

The fuzzy topology defined on $\widetilde{A}$ is $\tilde{T}$ = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$ } Then $\widetilde{E}$ and $\widetilde{F}$ are fuzzy δ-separated sets in $\widetilde{A}$ but not fuzzy R-separated sets in $\widetilde{A}$, Since: $\mu_{\boldsymbol{\delta}cl(\widetilde{E})}(x) = \widetilde{B}$, $\mu_{\boldsymbol{\delta}cl(\widetilde{F})}(x) = \widetilde{C}$

min{ $\mu_{\boldsymbol{\delta}cl(\widetilde{E})}(x)$, $\mu_{\widetilde{F}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\boldsymbol{\delta}cl(\widetilde{F})}(x)$, $\mu_{\widetilde{E}}(x)$}= $\mu_{\tilde{\emptyset}}(x)$, Hence $\widetilde{E}$ and $\widetilde{F}$ are fuzzy δ-separated in $\widetilde{A}$, But $\mu_{\boldsymbol{R}cl(\widetilde{E})}(x) = \widetilde{A}$, $\mu_{\boldsymbol{R}cl(\widetilde{F})}(x) = \widetilde{A}$

min{ $\mu_{\boldsymbol{R}cl(\widetilde{E})}(x)$, $\mu_{\widetilde{F}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\boldsymbol{R}cl(\widetilde{F})}(x)$, $\mu_{\widetilde{E}}(x)$} $\neq \mu_{\tilde{\emptyset}}(x)$

Then $\widetilde{E}$ and $\widetilde{F}$ are not fuzzy R-separated sets in $\widetilde{A}$.

3.12 Theorem

If $(\tilde{A}, \tilde{T})$ is a fuzzy topological space,and if :-

1. $\tilde{B}$ and $\tilde{C}$ are fuzzy θ-separated sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets. (resp fuzzy γ-separated sets , fuzzy Δ-separated sets.)
2. $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy γ-separated sets (resp fuzzy Δ-separated sets).
3. $\tilde{B}$ and $\tilde{C}$ are fuzzy R-separated sets in $\tilde{A}$ then $\tilde{B}$ and $\tilde{C}$ are fuzzy γseparated sets (resp fuzzy Δ-separated sets).

Proof: Obvious

3.13 Remark: The converse of theorem (3.12) is not true in general

3.14 Remark: Figure - 1 – illustrates the relation between fuzzy δ-separated set and some types of fuzzy separated sets.

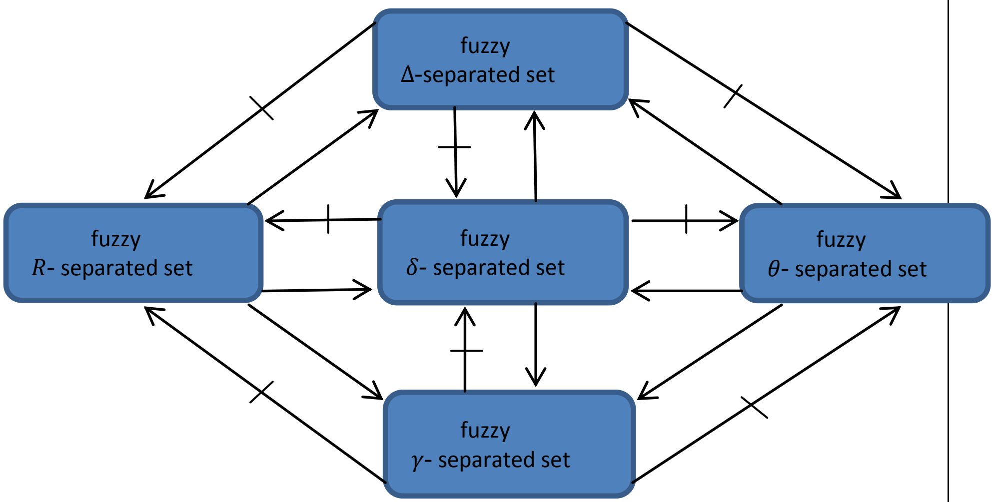

Figure - 1 –

4 . fuzzy δ-connected space

4.1 Definition

A fuzzy topological space $(\tilde{A}, \tilde{T})$ is said to be fuzzy δ-connected if there is no proper non-empty maximal fuzzy δ-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\}$ $\forall x \in X$ If $(\tilde{A}, \tilde{T})$ is not fuzzy δ-connected then is said to be fuzzy δ-disconnected spaces .

4.2 Theorem

A fuzzy topological space (Ã ,T̃) is fuzzy δ-connected if and only if there exist no non-empty fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \}$ and $\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)$

Proof :

(⇒) suppose that (Ã ,T̃) is fuzzy δ-connected space

Suppose that there exist non empty fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \}$ and $\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)$ Since $\tilde{E}$ and $\tilde{F}$ are fuzzy δ-closed sets in Ã and $\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)$ then $\tilde{E}$ and $\tilde{F}$ are fuzzy δ-separated sets in Ã since $\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \}$ then (Ã ,T̃) is fuzzy δ-disconnected , which is a contradiction .

(⇐) Suppose that there exist no non-empty fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \}$ and $\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)$

Suppose that (Ã ,T̃) is fuzzy δ-disconnected space , then this implies that there exist non-empty maximal fuzzy δ-separated sets $\tilde{B}$ and $\tilde{C}$ in Ã such that $\mu_{\tilde{A}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \}$ since $\tilde{B}$ and $\tilde{C}$ are fuzzy δ-separated in Ã , Then $\min\{ \mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{ \mu_{\delta\mathrm{cl}(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

implies that $\mu_{\tilde{C}}(x) \leq [\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c$ and $\mu_{\tilde{B}}(x) \leq [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$

since $\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\} \leq \max\{[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c, [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$

Then $\mu_{\tilde{A}}(x) = \max\{[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c, [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c \}$

Then $\mu_{\tilde{A}}(x) = \min [\mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$, $[\mu_{\tilde{A}}(x)]^c = \min [\mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\delta\mathrm{cl}(\tilde{C})}(x)]$

$\mu_{\tilde{\emptyset}}(x) = \min [\mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\delta\mathrm{cl}(\tilde{C})}(x)]$

Let $\mu_{\delta\mathrm{cl}(\tilde{C})}(x) = \mu_{\tilde{E}}(x)$ and $\mu_{\delta\mathrm{cl}(\tilde{B})}(x) = \mu_{\tilde{F}}(x)$, then $\min \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{\emptyset}}(x)$

And $\max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \max \{\mu_{\delta\mathrm{cl}(\tilde{C})}(x), \mu_{\delta\mathrm{cl}(\tilde{B})}(x) \}$

$\max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\delta\mathrm{cl}(\max\{\mu_{\tilde{C}}(x), \mu_{\tilde{B}}(x)\})}(x)$, $\max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\delta\mathrm{cl}(\tilde{A})}(x)$

$\max \{ \mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x) \} = \mu_{\tilde{A}}(x)$ which is a contradiction Hence (Ã ,T̃) is fuzzy δ-connected space. ■

4.3 Corollary

A fuzzy topological space (Ã ,T̃) is fuzzy δ-connected if and only if there exist no non-empty fuzzy δ-open sets $\widetilde{G}$ and $\widetilde{H}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\}$ and $\min\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Proof :

(⇒) suppose that (Ã ,T̃) is fuzzy δ-connected space

Suppose that there exist non empty fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\}$ and $\min\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Implies that $[\mu_{\tilde{A}}(x)]^c = \min\{[\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c\}$ and $\max\{[\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c\} = [\mu_{\tilde{\emptyset}}(x)]^c$ then

$\mu_{\tilde{\emptyset}}(x) = \min\{[\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c\}$ and $\max\{[\mu_{\tilde{G}}(x)]^c, [\mu_{\tilde{H}}(x)]^c\} = \mu_{\tilde{A}}(x)$

Let $[\mu_{\tilde{G}}(x)]^c = \mu_{\tilde{E}}(x)$ and $[\mu_{\tilde{H}}(x)]^c = \mu_{\tilde{F}}(x)$, Implies that that there exist no non-empty fuzzy δ-closed sets $\tilde{E}$ and $\tilde{F}$ in Ã such that $\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x)\}$ and $\min\{\mu_{\tilde{E}}(x), \mu_{\tilde{F}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Then (Ã ,T̃) is fuzzy δ-disconnected space which is a contradiction

(⇐) Suppose that there exist no non-empty fuzzy δ-open sets $\widetilde{G}$ and $\widetilde{H}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\}$ and $\min\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

Suppose that (Ã ,T̃) is fuzzy δ-disconnected space , then this implies that there exist non-empty maximal fuzzy δ-separated sets $\widetilde{B}$ and $\tilde{C}$ in Ã such that

$\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\}$ since $\widetilde{B}$ and $\tilde{C}$ are fuzzy δ-separated in Ã then

$\min\{\mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\tilde{C}}(x)\} = \mu_{\tilde{\emptyset}}(x)$ and $\min\{\mu_{\delta\mathrm{cl}(\tilde{C})}(x), \mu_{\tilde{B}}(x)\} = \mu_{\tilde{\emptyset}}(x)$

implies that $\mu_{\tilde{C}}(x) \leq [\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c$ and $\mu_{\tilde{B}}(x) \leq [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$

since $\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x)\} \leq \max\{[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c, [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$

Then $\mu_{\tilde{A}}(x) = \max\{[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c, [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c\}$,Let $[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c = \mu_{\tilde{G}}(x)$ and $[\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c = \mu_{\tilde{H}}(x)$

Then $\mu_{\tilde{A}}(x) = \max\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\}$, $\min\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\} = \min\{[\mu_{\delta\mathrm{cl}(\tilde{B})}(x)]^c, [\mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c\}$

$\min\{\mu_{\tilde{G}}(x), \mu_{\tilde{H}}(x)\} = \max[\mu_{\delta\mathrm{cl}(\tilde{B})}(x), \mu_{\delta\mathrm{cl}(\tilde{C})}(x)]^c$

$\min\{\ \mu_{\tilde{G}}(x)\ ,\ \mu_{\tilde{H}}(x)\}=[\mu_{\delta\text{cl}(\max\{\mu_{\tilde{C}}(x)\ ,\ \mu_{\tilde{B}}(x)\})}(x)]^c$

$\min\{\ \mu_{\tilde{G}}(x)\ ,\ \mu_{\tilde{H}}(x)\}=[\ \mu_{\delta\text{cl}(\tilde{A})}(x)\ ]^c = [\mu_{\tilde{A}}(x)]^c = \mu_{\tilde{\emptyset}}(x)$

which is a contradiction , hence (Ã ,T̃) is fuzzy δ-connected space. ■

4.4 Definition

A fuzzy topological space (Ã ,T̃) is said to be :

1. fuzzy R-connected if there is no proper non-empty maximal fuzzy R-separated sets $\tilde{B}$ and $\tilde{C}$ in Ã such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}\ \ \forall\ x \in X$
   If (Ã ,T̃) is not fuzzy R-connected then is said to be fuzzy R-disconnected spaces.

2. fuzzy Δ-connected if there is no proper non-empty maximal fuzzy Δ-separated sets $\tilde{B}$ and $\tilde{C}$ in Ã such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}\ \ \forall\ x \in X$
   If (Ã ,T̃) is not fuzzy Δ-connected then is said to be fuzzy Δ-disconnected spaces .

3. fuzzy γ-connected if there is no proper non-empty maximal fuzzy γ-separated sets $\tilde{B}$ and $\tilde{C}$ in Ã such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}\ \forall\ x \in X$
   If (Ã ,T̃) is not fuzzy γ-connected then is said to be fuzzy γ-disconnected spaces .

4. fuzzy θ-connected if there is no proper non-empty maximal fuzzy θ-separated sets $\tilde{B}$ and $\tilde{C}$ in Ã such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}\quad \forall\ x \in X$
   If (Ã ,T̃) is not fuzzy θ-connected then is said to be fuzzy θ-disconnected spaces .

4.5 Theorem

Let (Ã ,T̃) be a fuzzy topological space then:-

1. Every fuzzy δ-connected space is fuzzy R-connected space (resp. fuzzy θ-connected space) .

2. Every fuzzy γ-connected space is fuzzy R-connected space,(resp. fuzzy δ-connected space , fuzzy θ-connected space).

3. Every fuzzy Δ-connected space is fuzzy R-connected space,(resp. fuzzy θ-connected space , fuzzy δ-connected space).

Proof : **(1)** let (Ã ,T̃) is fuzzy δ-connected space

Suppose that (Ã, T̃) is fuzzy R-disconnected space , Then this implies that there exist non-empty maximal fuzzy R-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$ such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}$

Then there exist non-empty maximal fuzzy δ-separated sets $\tilde{B}$ and $\tilde{C}$ in $\tilde{A}$

Such that $\mu_{\tilde{A}}(x) = \max\{\ \mu_{\tilde{B}}(x)\ ,\ \mu_{\tilde{C}}(x)\ \}$ Implies that (Ã ,T̃) is fuzzy δ-disconnected

Which is a contradiction, hence (Ã, T̃) is fuzzy R-connected space. ■

Proof: - (2) and (3) similarly (1)

4.6 Remark

The converse of theorem (4.5) is not true in general as following examples shows **.**

4.7 Examples

1) Let X = { a , b } and $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\widetilde{E}$, are fuzzy subset in $\widetilde{A}$ where $\widetilde{A}$ = { (a , 0.8) , (b , 0.7) } , $\widetilde{B}$ = { (a , 0.0) , (b , 0.7) } , $\widetilde{C}$ = { (a , 0.5) , (b , 0.0) } $\widetilde{D}$ = { (a , 0.5) , (b , 0.7) } , $\widetilde{E}$ = { (a , 0.8) , (b , 0.0) } The fuzzy topology defined on $\widetilde{A}$ is $\tilde{T}$ = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$ } then FδC = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\widetilde{E}$ } and FRC = { $\emptyset$, $\widetilde{A}$, $\widetilde{E}$ } , $\widetilde{B}$ and $\widetilde{E}$ are fuzzy δ-separated sets in $\widetilde{A}$ Since: $\mu_{\delta\mathrm{cl}(\widetilde{B})}(x) = \widetilde{B}$, $\mu_{\delta\mathrm{cl}(\widetilde{E})}(x) = \widetilde{E}$

min{ $\mu_{\delta\mathrm{cl}(\widetilde{B})}(x)$, $\mu_{\widetilde{E}}$ (x)}= $\mu_{\widetilde{\emptyset}}(x)$ and min{ $\mu_{\delta\mathrm{cl}(\widetilde{E})}(x)$, $\mu_{\widetilde{B}}$ (x)}= $\mu_{\widetilde{\emptyset}}(x)$

and $\mu_{\widetilde{A}}$ (x) = max { $\mu_{\widetilde{B}}$ (x) , $\mu_{\widetilde{E}}$ (x) }, hence ($\tilde{A}$,$\tilde{T}$) is fuzzy δ-disconnected space

But ($\tilde{A}$,$\tilde{T}$) is fuzzy R-connected space since : $\mu_{R\mathrm{cl}(\widetilde{B})}(x) = \tilde{A}$, $\mu_{R\mathrm{cl}(\widetilde{E})}(x) = \widetilde{E}$

min{ $\mu_{R\mathrm{cl}(\widetilde{B})}(x)$, $\mu_{\widetilde{E}}$ (x)}$\neq$ $\mu_{\widetilde{\emptyset}}(x)$ hence $\widetilde{B}$, $\widetilde{E}$ are not fuzzy R-separated .

- FθC = { $\emptyset$, $\widetilde{A}$, $\widetilde{E}$ }, then ($\tilde{A}$,$\tilde{T}$) is fuzzy θ-connected space since :

$\mu_{\theta\mathrm{cl}(\widetilde{B})}(x) = \tilde{A}$, $\mu_{\theta\mathrm{cl}(\widetilde{E})}(x) = \widetilde{E}$, min{ $\mu_{\theta\mathrm{cl}(\widetilde{B})}(x)$, $\mu_{\widetilde{E}}$ (x)}$\neq$ $\mu_{\widetilde{\emptyset}}(x)$

hence $\widetilde{B}$, $\widetilde{E}$ are not fuzzy R-separated .

4.8 Example

2) Let X = { a , b } and $\widetilde{B}_1$, $\widetilde{B}_2$, $\widetilde{B}_3$, $\widetilde{B}_4$, $\widetilde{B}_5$, $\widetilde{B}_6$, $\widetilde{B}_7$, $\widetilde{B}_8$, $\widetilde{B}_9$, $\widetilde{B}_{10}$, $\widetilde{B}_{11}$, $\widetilde{B}_{12}$, $\widetilde{B}_{13}$, $\widetilde{B}_{14}$, $\widetilde{B}_{15}$, $\widetilde{B}_{16}$, $\widetilde{B}_{17}$ are fuzzy subset in $\widetilde{A}$ where $\widetilde{A}$ = { (a , 0.8) , (b , 0.9) }$\widetilde{B}_1$={ (a , 0.6) , (b , 0.0) } , $\widetilde{B}_2$ ={ (a , 0.7) , (b , 0.1) } $\widetilde{B}_3$={ (a , 0.1) , (b , 0.9) } , $\widetilde{B}_4$ ={ (a , 0.0) , (b , 0.8) } , $\widetilde{B}_5$= { (a , 0.1) , (b , 0.0) } , $\widetilde{B}_6$= { (a , 0.6) , (b , 0.9) } , $\widetilde{B}_7$={ (a , 0.6) , (b , 0.8) } , $\widetilde{B}_8$={ (a , 0.1) , (b , 0.1) } $\widetilde{B}_9$={ (a , 0.0) , (b , 0.1) } , $\widetilde{B}_{10}$={ (a , 0.6) , (b , 0.1) } ,$\widetilde{B}_{11}$={ (a , 0.7) , (b , 0.9) } , $\widetilde{B}_{12}$={ (a , 0.7) , (b , 0.8) } , $\widetilde{B}_{13}$={ (a , 0.1) , (b , 0.8) } , $\widetilde{B}_{14}$={ (a , 0.8) , (b , 0.1) } $\widetilde{B}_{15}$={ (a , 0.8) , (b , 0.8) } , $\widetilde{B}_{16}$={ (a , 0.8) , (b , 0.0) }, $\widetilde{B}_{17}$={ (a , 0.0) , (b , 0.9) }

The fuzzy topology defined on $\widetilde{A}$ is $\tilde{T}$={$\emptyset$, $\tilde{A}$, $\widetilde{B}_1$, $\widetilde{B}_2$, $\widetilde{B}_3$, $\widetilde{B}_4$, $\widetilde{B}_5$, $\widetilde{B}_6$, $\widetilde{B}_7$, $\widetilde{B}_8$,$\widetilde{B}_9$, $\widetilde{B}_{10}$, $\widetilde{B}_{11}$, $\widetilde{B}_{12}$, $\widetilde{B}_{13}$, $\widetilde{B}_{14}$, $\widetilde{B}_{15}$} then FγC={$\emptyset$, $\tilde{A}$, $\widetilde{B}_1$, $\widetilde{B}_2$, $\widetilde{B}_3$, $\widetilde{B}_4$, $\widetilde{B}_5$, $\widetilde{B}_8$, $\widetilde{B}_9$, $\widetilde{B}_{11}$, $\widetilde{B}_{12}$, $\widetilde{B}_{13}$, $\widetilde{B}_{14}$, $\widetilde{B}_{15}$, $\widetilde{B}_{16}$, $\widetilde{B}_{17}$} and FRC={$\emptyset$, $\tilde{A}$, $\widetilde{B}_1{}^c$, $\widetilde{B}_2{}^c$, $\widetilde{B}_3{}^c$, $\widetilde{B}_4{}^c$, $\widetilde{B}_5{}^c$, $\widetilde{B}_8{}^c$, $\widetilde{B}_9{}^c$, $\widetilde{B}_{11}{}^c$, $\widetilde{B}_{12}{}^c$, $\widetilde{B}_{13}{}^c$, $\widetilde{B}_{14}{}^c$, $\widetilde{B}_{15}{}^c$} , $\widetilde{B}_{16}$ and $\widetilde{B}_{17}$ are fuzzy γ-separated sets in $\widetilde{A}$ Since: $\mu_{\gamma\mathrm{cl}(\widetilde{B}_{16})}(x) = \widetilde{B}_{16}$, $\mu_{\gamma\mathrm{cl}(\widetilde{B}_{17})}(x) = \widetilde{B}_{17}$

min{ $\mu_{\gamma cl(\widetilde{B}_{16})}(x)$, $\mu_{\widetilde{B}_{17}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\gamma cl(\widetilde{B}_{17})}(x)$, $\mu_{\widetilde{B}_{16}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$

and $\mu_{\tilde{A}}$ (x) = max { $\mu_{\widetilde{B}_{16}}$ (x) , $\mu_{\widetilde{B}_{17}}$ (x) }, hence (Ã ,Ť) is fuzzy δ-disconnected space

But (Ã ,Ť) is fuzzy R-connected space since :

$\mu_{Rcl(\widetilde{B}_{16})}(x) = \tilde{B}_4{}^c$, $\mu_{Rcl(\widetilde{B}_{17})}(x) = \widetilde{B}_2{}^c$

min{ $\mu_{Rcl(\widetilde{B}_{16})}(x)$, $\mu_{\widetilde{B}_{17}}$ (x)}≠ $\mu_{\tilde{\emptyset}}(x)$ hence $\tilde{B}_{16}$, $\widetilde{B}_{17}$ are not fuzzy R-separated .

- FδC={$\emptyset$, $\tilde{A}$, $\widetilde{B}_1$, $\widetilde{B}_2$, $\tilde{B}_3$, $\tilde{B}_4$, $\tilde{B}_5$, $\tilde{B}_6$, $\tilde{B}_7$, $\tilde{B}_8$,$\widetilde{B}_9$, $\tilde{B}_{10}$, $\tilde{B}_{11}$, $\tilde{B}_{12}$, $\tilde{B}_{13}$, $\tilde{B}_{14}$, $\tilde{B}_{15}$}, then

(Ã ,Ť) is fuzzy δ-connected space since : $\mu_{\delta cl(\widetilde{B}_{16})}(x) = \tilde{B}_{14}$, $\mu_{\delta cl(\widetilde{B}_{17})}(x) = \tilde{B}_3$

min{ $\mu_{\delta cl(\widetilde{B}_{16})}(x)$, $\mu_{\widetilde{B}_{17}}$ (x)}≠ $\mu_{\tilde{\emptyset}}(x)$ hence $\widetilde{B}_{16}$, $\widetilde{B}_{17}$ are not fuzzy R-separated .

- FθC={$\emptyset$, $\tilde{A}$, $\widetilde{B}_2$, $\tilde{B}_4$, $\tilde{B}_5$, $\tilde{B}_8$,$\widetilde{B}_9$, $\tilde{B}_{11}$, $\tilde{B}_{12}$, $\tilde{B}_{13}$, $\tilde{B}_{14}$, $\tilde{B}_{15}$}, Then (Ã ,Ť) is fuzzy θ-connected space since $\mu_{\theta cl(\widetilde{B}_{16})}(x) = \tilde{B}_{14}$, $\mu_{\theta cl(\widetilde{B}_{17})}(x) = \tilde{B}_{11}$, min{ $\mu_{\delta cl(\widetilde{B}_{16})}(x)$, $\mu_{\widetilde{B}_{17}}$ (x)}≠ $\mu_{\tilde{\emptyset}}(x)$

hence $\widetilde{B}_{16}$, $\widetilde{B}_{17}$ are not fuzzy R-separated .

4.9 Example

3) Let X = { a , b } and $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\tilde{G}$, $\widetilde{H}$, $\tilde{I}$, are fuzzy subset in $\widetilde{A}$ where
$\widetilde{A}$ = { (a , 0.7) , (b , 0.8) } , $\widetilde{B}$ = { (a , 0.6) , (b , 0.0) } , $\widetilde{C}$ = { (a , 0.0) , (b , 0.7) }
$\widetilde{D}$ = { (a , 0.0) , (b , 0.6) } , $\tilde{E}$ = { (a , 0.5) , (b , 0.0) } , $\tilde{F}$ = { (a , 0.6) , (b , 0.7) } , $\widetilde{G}$ = { (a , 0.5) , (b , 0.7) } , $\widetilde{H}$ = { (a , 0.7) , (b , 0.0) } , $\tilde{I}$ = { (a , 0.0) , (b , 0.8) }

The fuzzy topology defined on $\widetilde{A}$ is Ť = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\tilde{C}$, $\widetilde{D}$, $\tilde{F}$, $\tilde{G}$ } ,

Then FRSO($\widetilde{A}$) ={$\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{H}$, $\tilde{I}$ }, FΔC($\widetilde{A}$) = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}^c$, $\widetilde{C}^c$, $\widetilde{H}^c$, $\tilde{I}^c$ } and FRC($\widetilde{A}$) = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}^c$, $\widetilde{C}^c$ } , $\widetilde{H}$ and $\tilde{I}$ are fuzzy Δ-separated sets in $\widetilde{A}$

Since: $\mu_{\Delta cl(\widetilde{H})}(x) = \widetilde{H}$, $\mu_{\Delta cl(\tilde{I})}(x) = \tilde{I}$

min{ $\mu_{\Delta cl(\widetilde{H})}(x)$, $\mu_{\tilde{I}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$ and min{ $\mu_{\Delta cl(\tilde{I})}(x)$, $\mu_{\widetilde{H}}$ (x)}= $\mu_{\tilde{\emptyset}}(x)$

And $\mu_{\tilde{A}}$ (x) = max { $\mu_{\widetilde{H}}$ (x) , $\mu_{\tilde{I}}$ (x) } hence (Ã ,Ť) is fuzzy Δ-disconnected space

But (Ã ,Ť) is fuzzy R-connected space since : $\mu_{Rcl(\widetilde{H})}(x) = \tilde{B}^c$, $\mu_{Rcl(\tilde{I})}(x) = \widetilde{C}^c$

min{ $\mu_{Rcl(\widetilde{H})}(x)$, $\mu_{\tilde{I}}$ (x)}≠ $\mu_{\tilde{\emptyset}}(x)$ hence $\widetilde{B}$, $\tilde{E}$ are not fuzzy R-separated

- FδC = { $\emptyset$, $\widetilde{A}$, $\widetilde{B}$, $\tilde{C}$, $\widetilde{D}$, $\tilde{E}$, $\tilde{F}$, $\widetilde{G}$ } then (Ã ,Ť) is fuzzy δ-connected space since :

$\mu_{\delta cl(\widetilde{H})}(x) = \tilde{A}$, $\mu_{\delta cl(\tilde{I})}(x) = \tilde{A}$

min{ $\mu_{\delta cl(\widetilde{H})}(x)$, $\mu_{\tilde{I}}$ (x)}≠ $\mu_{\tilde{\emptyset}}(x)$ hence $\widetilde{H}$, $\tilde{I}$ are not fuzzy δ-separated .

- $F\theta C = \{ \emptyset , \widetilde{A} \}$, then $(\tilde{A} ,\tilde{T})$ is fuzzy θ-connected space since :

$\mu_{\theta \mathrm{cl}(\widetilde{H})}(x) = \tilde{A}$, $\mu_{\theta \mathrm{cl}(\tilde{I})}(x) = \tilde{A}$

$\min\{ \mu_{\theta \mathrm{cl}(\widetilde{H})}(x) , \mu_{\tilde{I}} (x)\} \neq \mu_{\tilde{\emptyset}}(x)$ hence $\widetilde{H}$, $\tilde{I}$ are not fuzzy θ-separated .

4.10 Remark: Figure - 2 – illustrates the relation between fuzzy δ-connected space and some types of fuzzy connected space .

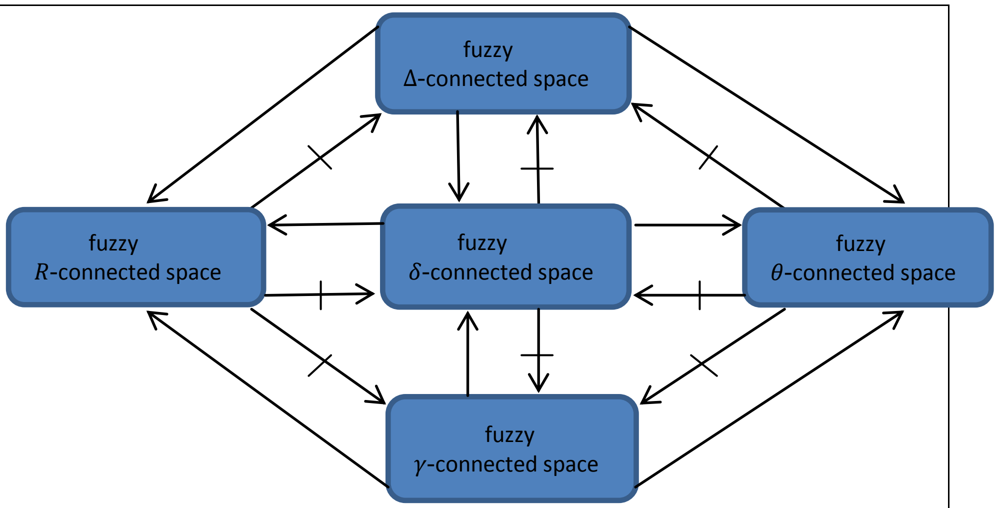

Figure - 2 –

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