# A survey of probabilistic reliability calculation for computer communication networks

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**Abstract.** The topological reliability of communication network is a key point of the present network reliability researches, which is the study of network reliability using graph theory. The study of network reliability can be also considered as first step of the design of "good" communication network. This paper presents a survey of analytical methods "probabilistic methods" used for network reliability calculation. We are interested in obtaining an analysis quickly, in particular, quickly enough to be able to make use of this analysis in real time for network management. The main reliability algorithms are presented, analyzed based on complexity and case studies where they can be applied.

**Keyword:** computer network, reliability, probability, topology, graph theory.

## 1 INTRODUCTION

The reliability is the probability of no failure within a given operating period for a communication link or any electrical, electronic device. Network reliability has long been a practical issue, and will remain so for years, since networks have entered an era of Quality of Service (QoS). In managing a Computer Network and planning topological modifications, it is important to be able to determine reliability measures quickly. In the most general case, we would like to determine a precise relationship between the failure of network components and the amount of traffic the network can handle. Such analysis is usually complex and time consuming as it involves not only a combinatorial analysis of the states arising from failed components, but also an analysis of the routing within the network as presented by Shooman (2002). Network reliability is an important step toward the design of robust survivable network for many important case studies for critical processes such in military, chemical industry, oil refinery, and many other applications (Mahmood, Al-Naima, & Uzunoglu, 2012).

Network reliability is a very important issue for network and electronic systems designers because a good reliability study will lead to (Jonczy, 2006):

- Increase the independency of the system on networks failures,
- Good design and analysis of networks led to increasing networks vulnerability due to component failures,
- The study of reliability will affect the choice of communication network protocols, and topology.

Many physical problems as computer networks, piping systems, and power grids can be modeled by a network. In the context of this work, the word network means a physical problem that can be modeled as a mathematical graph composed of nodes and links (directed or undirected) where the branches have associated physical parameters such as flow per minute, bandwidth, or Mega watts.

The study of network reliability has led to a huge body of literature. The calculation of network reliability in a probabilistic context has long been an issue of practical and academic importance. Conventional approaches such as determination of bounds (Konak, 2007) sums of disjoint products algorithms (Balan, 2003), Monte Carlo evaluations (Armando, Leonidas, & Vladimiro, 2007), studies of the reliability polynomials (Chang & Shrock, 2003), etc., only provide approximations when the network's size increases, even when nodes do not fail and all edges have the same reliability (p).

If we focus on communication between a pair of nodes where (s) is the source node and (t) is the target node, then successful operation is defined as the presence of one or more operating paths between (s) and (t). This is called the two-terminal problem, and the probability of successful communication between (s) and (t) is called two-terminal reliability (Suri & Bhushan, 2008).

The all-terminal reliability (this is sometimes termed overall network reliability) problem is somewhat more difficult than the two-terminal reliability problem. Essentially, we must modify the two-terminal problem to account for all-terminal pairs. All-terminal reliability is the probability that a set of operational edges provides communication paths between every pair of nodes ($n_i, n_j$ with $n_i \neq n_j$). While in the case of two-terminal problem one pair (s, t) is considered (Altiparmak, Dengiz, & Smith 2009). One can define a more general concept of k-terminal reliability, where k terminals must be connected. If k=2, we have two-terminal reliability, while in the case k=all terminals, we have all-terminal reliability. Thus k-terminal reliability can be viewed as a more general concept (Kuo, Yeh, & Lin, 2007; Yeh, Lu, & Kuo, 2002). This paper introduces the main methods of network reliability calculation. We are interested especially into 2-terminal reliability, but same methods can be used for all-terminals problem.

By considering the probabilistic approach, in which the network is represented by graph G = (V, E), where V is a set of nodes (also called vertices) and E is a set of directed or undirected edges (or links). Each of which having a probability $p_n$ (for nodes) or $p_e$ (for edges) to operate correctly. Failures of the different constituents are assumed to occur at random, and to be statistically independent events. In the most general model, both nodes and links can fail but for simplified model nodes may be considered as perfect by the use of redundant materials.

## 2 METHODS OF RELIABILITY EVALUATION

Reliability calculation is mainly based into mathematical development of graph theory and probability theory. Different classifications of methods used to find the reliability exist in many previous works. Methods can be classified into polynomial and non-polynomial, or into exact and approximated methods, and so on. The classification mentioned by this work looks like the best covering partition of methods used to evaluate reliability as fallow:

### 2.1 State space enumeration method (SSE)

The simplest means of evaluating the two-terminal reliability of a network is to enumerate all possible combinations where each of the (e) edges can be good or bad, resulting in $2^e$ combinations (Shooman, 2002). Each of these combinations of good and bad edges can be treated as an event $E_i$, These events are all mutually exclusive (disjoint), and the reliability expression is simply the probability of the union of the sub-set of these events that contain a path between (s) and (t) as presented in Eq. (1)

$$R_{st} = P(E_1 + E_2 + \cdots), \tag{1}$$

since each of these events is mutually exclusive, the probability of the union becomes the sum of the individual event probabilities.

$$R_{st} = P(E_1) + P(E_2) + \cdots \quad . \tag{2}$$

As case study, the network in fig. (1) is considered. There are five nodes connected by six links, each one of reliability of 0.9 (90%). Links probabilities of good work (which is the link reliability) are taken to be equals in this example, but in general each link can have a different probability. The reliability to be computed is the reliability $R_{(a\rightarrow c)}$ which is the two-terminals reliability for source node (a) and destination (c). Two event types can be produced; the first is "good event- G" when there at least one path or route between the source node and the destination. The second is when no path is found between the source and the destination which is the "Bad event- B". The solution fount by collecting all good events and applying Eq. (2) will give the reliability.

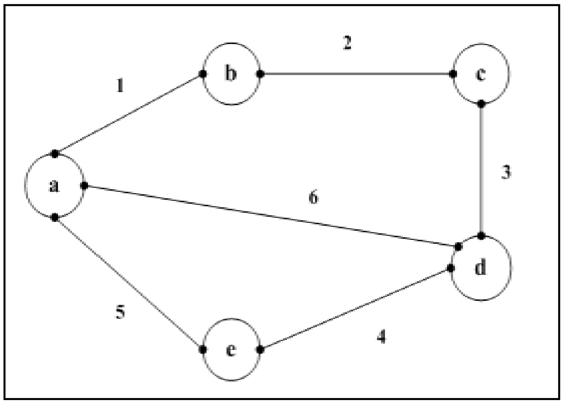


Fig.1. Connected network case study

From the probability principal, the number of all events can be computed by Eq. (3), where n represent links number in the network, and f the number of failed links.

$$\text{Number of Events} = C_f^n = \frac{n!}{(n-f)!\times f!} \quad . \tag{3}$$

For example for 3-failed links, $= \frac{6!}{(6-3)!3!} = \frac{6\times5\times4\times3\times2\times1}{(3\times2\times1)(3\times2\times1)} = 5 \times 4 = 20$ events.

Total number of events for this case study = $2^6 = 64$ events listed below in table (1).

Table 1. State enumeration events table

| Number of failed links | Events: $x$ = Good link , $\bar{x}$ = failed link , G= Good event, B= bad event |
|---|---|
| All good $C_0^6 = 1$ event | E1=1 2 3 4 5 6 (G) |
| One failure $C_1^6 = 6$ events | E(2)=$\bar{1}\,2\,3\,4\,5\,6$(G), E(3)=$1\bar{2}\,3\,4\,5\,6$ (G), E(4)=$1\,2\,\bar{3}\,4\,5\,6$ (G), E(5)=$1\,2\,3\,\bar{4}\,5\,6$ (G), E(6)=$1\,2\,3\,4\,\bar{5}\,6$(G), E(7)=$1\,2\,3\,4\,5\,\bar{6}$ (G) |
| Two failures $C_2^6 = 15$ events | $\bar{1}\,\bar{2}\,3\,4\,5\,6$(G), $\bar{1}\,2\,\bar{3}\,4\,5\,6$ (B), $\bar{1}\,2\,3\,\bar{4}\,5\,6$(G), $\bar{1}\,2\,3\,4\,\bar{5}\,6$(G), $\bar{1}\,2\,3\,4\,5\,\bar{6}$(G), $1\,\bar{2}\,\bar{3}\,4\,5\,6$(B), $1\,\bar{2}\,3\,\bar{4}\,5\,6$ (G), $1\,\bar{2}\,3\,4\,\bar{5}\,6$ (G), $1\,\bar{2}\,3\,4\,5\,\bar{6}$(G), $12\bar{3}\bar{4}56$(G), $12\bar{3}4\bar{5}6$(G), $12\bar{3}45\bar{6}$(G), $123\bar{4}\bar{5}6$(G), $123\bar{4}5\bar{6}$(G), $1234\bar{5}\bar{6}$(G). |
| Three failures $C_3^6 = 20$ events | $\bar{1}\,\bar{2}\,\bar{3}\,4\,5\,6$(B),$\bar{1}\,\bar{2}\,3\,\bar{4}\,5\,6$(G),$\bar{1}\,\bar{2}\,3\,4\,\bar{5}\,6$(G),$\bar{1}\,\bar{2}\,3\,4\,5\,\bar{6}$(G),$\bar{1}\,2\,\bar{3}\,\bar{4}\,5\,6$(B),$\bar{1}\,2\,\bar{3}\,4\,\bar{5}\,6$(B),$\bar{1}\,2\,\bar{3}\,4\,5\,\bar{6}$(B),$\bar{1}\,2\,3\,\bar{4}\,\bar{5}\,6$(G),$\bar{1}\,2\,3\,\bar{4}\,5\,\bar{6}$(B),$\bar{1}\,2\,3\,4\,\bar{5}\,\bar{6}$(B),$1\,\bar{2}\,\bar{3}\,\bar{4}\,5\,6$(B),$1\,\bar{2}\,\bar{3}\,4\,\bar{5}\,6$(B), $1\,\bar{2}\,\bar{3}\,4\,5\,\bar{6}$(B),$1\,\bar{2}\,3\,\bar{4}\,\bar{5}\,6$(G),$1\,\bar{2}\,3\,\bar{4}\,5\,\bar{6}$(B),$1\,\bar{2}\,3\,4\,\bar{5}\,\bar{6}$(B),$1\,2\,\bar{3}\,\bar{4}\,\bar{5}\,6$(G),$1\,2\,\bar{3}\,\bar{4}\,5\,\bar{6}$(G),$1\,2\,\bar{3}\,4\,\bar{5}\,\bar{6}$(G), $1\,2\,3\,\bar{4}\,\bar{5}\,\bar{6}$(G). |
| 4- Failures $C_4^6 = 15$ | $\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,5\,6$(B),$\bar{1}\,\bar{2}\,\bar{3}\,4\,\bar{5}\,6$(B),$\bar{1}\,\bar{2}\,\bar{3}\,4\,5\,\bar{6}$(B),$\bar{1}\,\bar{2}\,3\,\bar{4}\,\bar{5}\,6$(G),$\bar{1}\,\bar{2}\,3\,\bar{4}\,5\,\bar{6}$(B),$\bar{1}\,\bar{2}\,3\,4\,\bar{5}\,\bar{6}$(B),$1\,\bar{2}\,\bar{3}\,\bar{4}\,\bar{5}\,6$(B),$1\,\bar{2}\,\bar{3}\,\bar{4}\,5\,\bar{6}$(B),$1\,2\,\bar{3}\,\bar{4}\,\bar{5}\,\bar{6}$(G),$\bar{1}\,2\,\bar{3}\,\bar{4}\,\bar{5}\,6$(B),$\bar{1}\,2\,\bar{3}\,\bar{4}\,5\,\bar{6}$(B),$\bar{1}\,2\,3\,\bar{4}\,\bar{5}\,\bar{6}$(B),$1\,\bar{2}\,3\,\bar{4}\,\bar{5}\,\bar{6}$(B),$1\,\bar{2}\,\bar{3}\,4\,\bar{5}\,\bar{6}$(B),$\bar{1}\,2\,\bar{3}\,4\,\bar{5}\,\bar{6}$(B). |
| 5- Failures $C_5^6 = 5$ | $\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,\bar{5}\,6$(B), $\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,5\,\bar{6}$(B), $\bar{1}\,\bar{2}\,\bar{3}\,4\,\bar{5}\,\bar{6}$(B), $\bar{1}\,\bar{2}\,3\,\bar{4}\,\bar{5}\,\bar{6}$(B), $\bar{1}\,2\,\bar{3}\,\bar{4}\,\bar{5}\,\bar{6}$(B), $1\,\bar{2}\,\bar{3}\,4\,\bar{5}\,\bar{6}$(B). |
| All-failures $C_6^6 = 1$ | $\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,\bar{5}\bar{6}$(B). |

Link (x) probability = $P(x) = 0.9$ (90% of time link is up), (4)

then $P(\bar{x}) = q(x) = 1 - 0.9 = 0.1$ (10% of time link is down). (5)

Combining Eqs. (2), (4), (5), and events in table 1 gives:

$$R_{ac} = 1 \times (0.9)^6 + 6 \times (0.9)^5 \times (0.1) + 13 \times (0.9)^4 \times (0.1)^2 + 9 \times (0.9)^3 \times (0.1)^3 + 2 \times (0.9)^2 \times (0.1)^4 + 0 \times (0.9)^1 \times (0.1)^5 + 0 \times (0.1)^6$$

$R_{ac} = 0.977751$ (97.7751% of good work). (6)

## 2.2 Cut-sets (CS) and tie-sets (TS) method

The reliability computing complexity can be reduced below the $2^e$ required for the enumeration method by the use of minimal cut sets (CS) and minimal tie sets (TS) methods.

TS are the groups of edges that form a path between (s) and (t). The term minimal implies that no node or edge is traversed more than once (loop free). If there are (i) tie sets between (s) and (t), then the reliability is given by the expansion of (Shooman, 2002) :

$$R_{st} = P(T_1 + T_2 + \cdots + T_i), \tag{7}$$

where $T_m$ is the TS number (m).

Similarly, one can focus on the minimal CS of a graph. CS are a group of edges that break all paths between (s) and (t) when they are removed from the graph. If CS is minimal, no subset is also a cut set. The reliability expression in terms of the (j) cut sets is given by the expansion of:

$$R_{st} = 1 - P(C_1 + C_2 + \cdots + C_j)\ , \tag{8}$$

where $C_m$ represents the CS number (m).

The complexity of the CS and TS methods depends on two factors: the order of complexity involved in finding the TS (or CS) and the order of complexity for the inclusion–exclusion expansion. Algorithms for finding the number of CS and TS are of polynomial complexity (Konak & Smith, 2006).

For the same case study given in fig. 1, TS method is used to calculate the reliability between node (a) and node (c). Table 2 lists all tie sets group for this topology. The formula of Poincare (the inclusion exclusion expansion) is used with Eqs. (4) or (5) to find the result:

$$P(A_1 + A_2 + \cdots + A_n) = \sum_{i=1}^{n} A_i - \sum_{1\le i\le j\le n} P\big(A_i A_j\big) + \cdots (-1)^{n+1} P(A_1 A_2 \dots A_{n)}. \tag{9}$$

Applying Eq. (9) and TS found in table 2 to calculate reliability $R_{ac}$:

$$R_{ac} = P(T_1) + P(T_2) + P(T_3) - [P(T_1T_2) + P(T_1T_3) + P(T_2T_3)] + [P(T_1T_2T_3)]$$

$$R_{ac} = P(12) + P(36) + P(345) - [P(1236) + P(12345) + P(3456)] + P(123456)$$

$$R_{ac} = 2p^2 + p^3 - [2p^4 + p^5] + p^6$$

$$R_{ac} = 2(0.9)^2 + (0.9)^3 - [2(0.9)^4 + (0.9)^5] + (0.9)^6$$

$$R_{ac} = 2.349 - 1.90269 + 0.531441$$

$$R_{ac} = 0.977751 \quad (97.7751\%), \tag{10}$$

which is exactly the same solution found in (6), but with less calculation complexity

Table 2. TS for the topology of figure (1).

| Tie Set | Links good in operation |
|---|---|
| T1 | 1 2 |
| T2 | 6 3 |
| T3 | 3 4 5 |

## 2.3 Graph transformation method (GT)

This method is based on transforming the network into a simpler network (or set of networks) by successively applying transformations. Transformations are based on three basic transformations as in fig. 2.

The series connection of fig.2 -a gives: $P_{ac} = P(1 \cdot 2) = P_1 \cdot P_2$.

The parallel connection of fig. 2-b gives: $P_{ab} = P(1+2) = 1 - q(1) \times q(2)$

Expansion about (5) for the fig. 2-c yields: $P_{ad} = P(5) \times P(G_1) + P(\bar{5}) \times P(G_2)$ Where $G_1$ is the graph with link (5) as short (node c and b coincide), and $G_2$ with link (5) failed (cut circuit).

The parallel, series and delta-star transformation are used by successive procedure to reduce the network to simpler network by node and link reduction technique (Rebaiaia, Ait-Kadi, & Merlano, 2009).

For the example of fig. 1 we have many simplifications based on the graph transformation as presented in fig. 3:

Link ($L_1$) and link ($L_2$) in series gives $L_7$ with probability equal to: $0.9 \times 0.9 = 0.81$ , $L_4$ and $L_5$ to get $L_8$ as in fig. 3-b.

$L_6$ and $L_8$ in parallel gives a link $L_9$ with probability equal to : $1 - 0.19 \times 0.1 = 0.981$ , as in fig. 3-c.

Link $L_3$ in series with $L_9$ gives $L_{10}$ with probability equal to : $0.981 \times 0.9 = 0.8829$ , as in fig. 3-d.

Finally $L_7$ in parallel with $L_{10}$ which gives the two-terminals reliability between node (a) and (c) presented in figure 3-e:

$$R_{ac} = 1 - 0.19 \times 0.1171 = 0.977751, \tag{11}$$

which is the same solution found in Eqs. (6) and (10) by previous methods but with less calculation complexity.

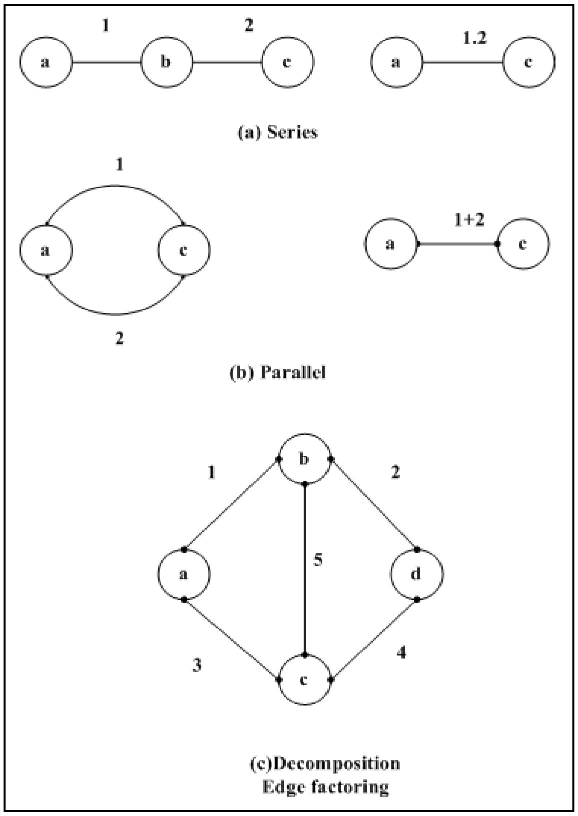


Fig. 2. Basic decomposition techniques

### 2.4 Approximated methods

There are many approximated methods which can be used to evaluate network reliability. The approach is to neglect some terms with small effect on reliability estimation. Reliability can be also bounded between two values representing a maximum and minimum bound. Two approximated methods are suggested in many previous works based on cut sets and tie sets methods. The first is the truncation Approximations method (TA). The inclusion-exclusion expansion in equation (9) becomes more complicated when the number of TS (or CS) becomes big, for that and for large network this technique is elaborated. A certain number of significant terms must be taking to evaluate the maximum and minimum bounds with accuracy.

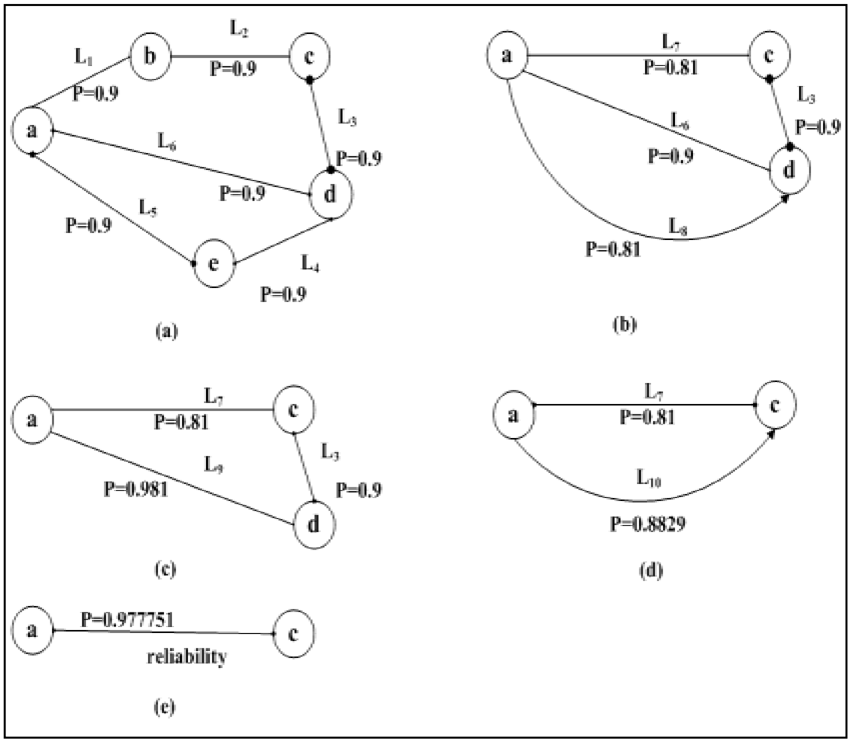


Fig. 3. Graph transformation of the case study

The second method is the subset approximation. The approximation is based on the exclusion of low probability cut sets or tie sets. Clearly, the occurrence probability of the lower order (fewer edges) cut sets is higher than the higher-order (more edges) ones.

With the advance of programming technique, it is easy today to implement mathematical equations and models. The approximation methods proposed previously are based on CS and TS methods. We propose to use approximation based on enumeration method for large network. The enumeration method is used today only with small network with few nodes and links. With this technique we can extend the use of this method to medium and large network.

## 3 CONCLUSION

Enumeration method is simple, used for small network because requires large number of operations. Increasing the number of failed links will decrease considerably the effect of corresponding event on the calculation of reliability which yields the possibility of using the approximation method as simplification procedure.

CS and TS method can be used for medium network size requiring less computation than the enumeration method. Approximation methods are widely used with this type of reliability calculation and found to be very useful. GT method can be used for all type of networks. It requires complicate algorithm but with reduced number of operations. The work today is focused over this method to find more and more of simplification based on new graph reduction method. Algorithms use different technique to recognize typical forms and use them to simplify network topology graph. Applying all methods for all-terminal reliability will be a simple repetition of the algorithm used in two-terminal. With the complication of network topologies the approximation methods represent a good technique to solve reliability calculation problem.

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