# The Effect of Using Distance Measures in Image Processing Systems

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**Abstract.** The effect of using distance measure for compare between textural images can be shown by use difference distance measure on deferent textural images ,result show that distance measure effected by more than one factor in addition to kind of image
Using distance and correlation measures can be done in order to show the effect of them

**Keywords:** Digital image processing, Distance measures ,MSE,MAPE,MPE,RMSD,MAE, low pass filter, high pass filter, image enhancement, image smoothing ,image Sharpening

## 1 INTRODUCTION

Digital image processing is a subset of the electronic domain wherein the Image is converted to an array of small integers called (pixels) representing Physical quantity such as scene radiance, stored in a digital memory, and processed by computer or other digital hardware.
Digital image processing, either as enhancement for human observers or performing autonomous analysis, offers advantages in cost, speed, and flexibility, and with the rapidly falling price and rising performance of personal computers it has become the dominant method in use.
Digital image processing has many advantages over analog image processing; it allows a much wider range of algorithms to be applied to the input image and by making changes on it in order to get out put image an image May be defined as a two-dimensional function f(x, y). Where( x and y )are spatial (plane) coordinates, and the amplitude of (f) at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point.
In our research we applied some filters (enhanced, smoothing and Sharpening) On some pictures with versos sizes and compare the output images by using some statistical measures Results shows that output images effected with many factors such that (size, filter kind, statistical compares measures)

## 2 AIM OF RESEARCH

The aim research is to compare high pass and low pass filters on input textural images with verses sizes by applying some statistical measures.

## 3 LOW PASS FILTER

These are used to emphasize large homogenous areas of similar tone and reduce the smaller detail. Low frequency areas are retained in the image resulting in a smoother appearance to the image

## 4 HIGH-PASS FILTERS

Allow high frequency areas to pass with the resulting image having greater detail resulting in a sharpened image.

## 5 IMAGE ENHANCEMENT

One of the strengths of image processing is that it gives us the ability to enhance the view of an area by manipulating the pixel values, thus making it easier for visual interpretation. There are several techniques which we can use to enhance an image, such as Contrast Stretching and Spatial Filtering

In image enhancement, the goal is to accentuate certain image features for subsequent analysis or for image display. Examples include contrast and edge enhancement, pseudo coloring, noise filtering, sharpening and magnifying. Image enhancement is useful in feature extraction, image analysis and visual information display. The enhancement process itself does not increase the inherent information display in the data. It simply emphasizes certain specified image characteristics. Enhancement algorithms are generally interactive and application dependent.

Enhancing the contrast of images is one of the major issues in image processing, especially backlit images. Contrast enhancement can be achieved by stretching the dynamic range of important objects in an image. There are many algorithms for contrast enhancement and among these

## 6 IMAGE SHARPENING.

Due to the reversibility of the Sobolev diffusion equations The most basic approach would be to run the Sobolev diffusion equations in the backward direction on a given image u0 and at some stopping time, say( $\tau > 0$), we declare u($\tau$ ) to be the sharpened version of u0. However, it is not clear in the approach how one would appropriately

Select the stopping time $\tau > 0$, as it would necessarily vary between images and applications domains.

## 7 IMAGE SMOOTHING

The effects of noise on images can be reduced by smoothing, that is, by replacing every pixel by a weighted average of its neighbors. This operation can be expressed by the following convolution (6):

$$\tau_{ij} = \sum_{i=a\ start}^{a\ end} \sum_{j=b\ start}^{b\ end} g_{(a,b)} f_{(i-a,j-b)}$$

where ($g$ ) is the convolution mask (or kernel or point-spread function) that lists the weights, ($f$) is the image, and

(a start, a end, b start, b end )delimit the domain of definition of the kernel, that is, the size of the neighborhood

involved in smoothing. The kernel is usually rotationally symmetric, as there is no reason to privilege, say, the pixels on the left of position i; j over those on the right

(-a start= a end, -b start= b end)

$$g_{(a,b)} = \gamma(r)$$

Were

$$r = \sqrt{a^2 + b^2}$$

is the distance from the center of the kernel to its element a; b. Thus, a rotationally symmetric kernel can be obtained by rotating a one-dimensional function (r) defined on the nonnegative around the origin of the plane

The purpose of smoothing is to reduce noise and improve the visual quality of the image. Often, smoothing is referred to as (filtering). There are two types of filters that have been found useful (spatial )and (temporal) for both static and dynamic images

## 8 IMAGE PROCESSING ALGORITHM

The adoptive algorithm depend on input image with $(M * N)$ pixels each one with gray level such that (fig 1) each pixel with

$P_{ij} \quad with \ i = 1, \dots, N \ \& \ j = 1, \dots, M$

Then by taking process image with (M-2*N-2) pixels each one with same original gray level except starting and ending rows and columns

$P^*_{ij} \quad with \ i = 1, \dots, N-2 \ \& \ j = 1, \dots, M-2$

By using process image and calculate the adoptive filter such that

$$\tau_{ij} = \sum_{i=2}^{N-1} \sum_{j=2}^{M-1} P^*_{ij} \, a_{ij}$$

Were

$P^*_{ij}$ *represent image processing pixels*

$\tau_{ij}$ *represent output pixels*

$a_{ij}$ *represent filter pixels*

The simplest filtering technique is the nine-point (3X3) window, The nine-point will take a (3X3) square of pixels (total of nine) and determine the number of counts in each pixel., this same operation can be repeated for the entire computer screen or restricted to a designated area.

The output image will be with $(\tau_{ij})$ as a pixel values

For adoptive filters $(a_{ij})$ will be

For Low pass enhancing filter

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 2 | 1 |
| 0 | 1 | 0 |

For Low pass Sharpening filter

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

For Low pass smoothing filter

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

For High pass enhancing filter

| | | |
|---|---|---|
| 0 | -1 | 0 |
| -1 | 5 | -1 |
| 0 | -1 | 0 |

For High pass Sharpening filter

| | | |
|---|---|---|
| -1 | -1 | -1 |
| -1 | 9 | -1 |
| -1 | -1 | -1 |

For High pass smoothing filter

| | | |
|---|---|---|
| 1 | -2 | 1 |
| -2 | 5 | -2 |
| 1 | -2 | 1 |

## 9 STATSITCAL MEASURES

Pixel values of input images will be changed after any process on it and pixel values of output images will have deferent values compare with input one, changes of pixel value will be low or high depend on some factors

In order to get changes on pixels value we have the following statistical measures

$$MSE = \frac{\sum_{i=2}^{N-2}\sum_{j=2}^{M-2}(\mathrm{P_{ij}} - \tau_{ij})^2}{(\mathrm{M}-2)(\mathrm{N}-2)}$$

$$\mathrm{MAPE} = \frac{\sum_{i=2}^{N-2}\sum_{j=2}^{M-2}\left(\frac{/\ \mathrm{P_{ij}} - \tau_{ij}\ /}{\tau_{ij}}\right)}{(\mathrm{M}-2)(\mathrm{N}-2)}$$

$$\mathrm{MPE} = \frac{\sum_{i=2}^{N-2}\sum_{j=2}^{M-2}\left(\frac{\mathrm{P_{ij}} - \tau_{ij}}{\tau_{ij}}\right)}{(\mathrm{M}-2)(\mathrm{N}-2)}$$

$$\mathrm{RMSD} = \sqrt{\frac{\sum_{i=2}^{N-2}\sum_{j=2}^{M-2}(\mathrm{P_{ij}} - \tau_{ij})^2}{(\mathrm{M}-2)(\mathrm{N}-2)-1}}$$

$$\mathrm{MAE} = \frac{\sum_{i=2}^{N-2}\sum_{j=2}^{M-2}/\ \mathrm{P_{ij}} - \tau_{ij}\ /}{(\mathrm{M}-2)(\mathrm{N}-2)}$$

Were

$P_{ij}$ represent input pixels value

$\tau_{ij}$ represent output pixels value

M x N represent size of image

## 10 EXPERIMENTAL TEXTURAL IMAGES

In order to determine research results we take (6) textural images as input data Such that fig(1)
D1=breast high
D2=breast low
D3=breast_low_2
D4=colon high
D5=colon low
D6=colon_low_2

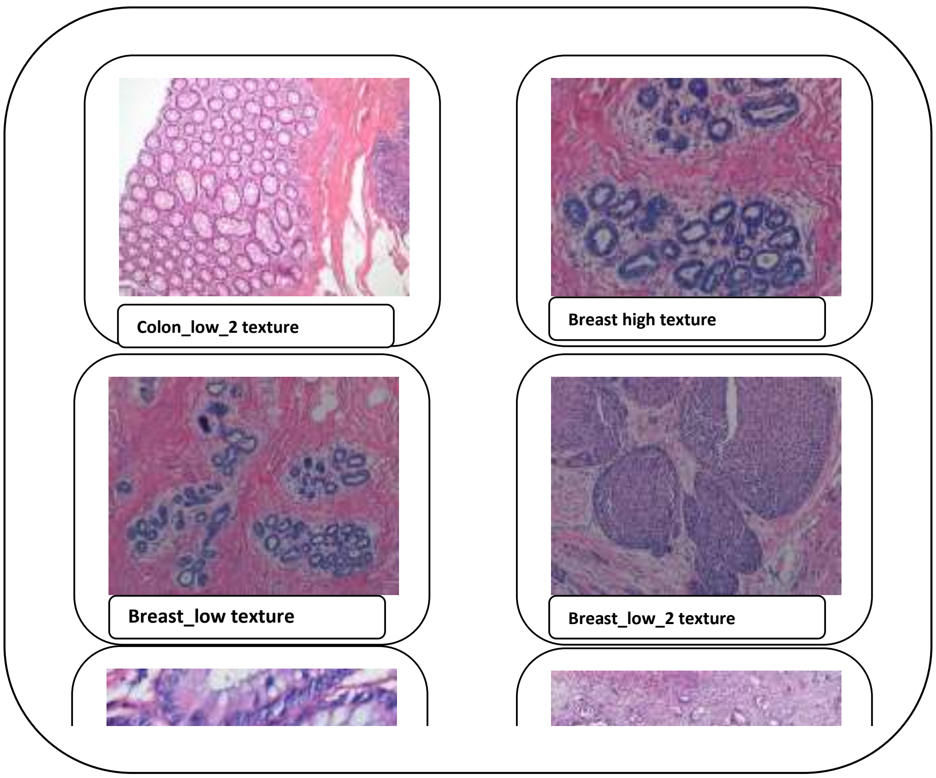



Fig (1)
Textural input data ref.(13)

Each textural image taken with (3) sizes such that
1st size with (640X480) pixels
2nd size with (320X240) pixels
3rd size with (160X120) pixels

## 11 EXPERIMENTAL RESULTS

In order to get the experimental results in this research all experimental textural images with all sizes was takes as input data with high and low pass filters the results can be showing in the following tables

Table (1)
Represent experimental results for (High pass filters)

| SIZE | IMAGE | Filter | M1 | M2 | M3 | M5 | M5 |
|---|---|---|---|---|---|---|---|
| 1st size | D1 | F1 | 1263.707 | 0.111457 | 6.65E-02 | 35.5489 | 20.19244 |
| | | F2 | 3143.696 | 0.17786 | 9.19E-02 | 56.06904 | 33.08687 |
| | | F3 | 897.7905 | 9.02E-02 | 3.73E-02 | 29.96335 | 17.29143 |
| | D2 | F1 | 238.5341 | 8.70E-02 | 1.25E-02 | 15.44465 | 10.98469 |
| | | F2 | 1291.003 | 0.207004 | 1.20E-02 | 35.93077 | 27.00098 |
| | | F3 | 92.8866 | 4.73E-02 | 2.44E-03 | 9.637832 | 6.158852 |
| | D3 | F1 | 545.8879 | 0.136962 | 1.79E-02 | 23.3644 | 17.21645 |
| | | F2 | 2412.77 | 0.299078 | 1.32E-03 | 49.12028 | 38.92848 |
| | | F3 | 241.2499 | 7.98E-02 | 5.28E-03 | 15.53232 | 10.05877 |
| | D4 | F1 | 716.7584 | 0.190706 | 3.15E-02 | 26.77252 | 20.57236 |
| | | F2 | 3111.161 | 0.400192 | -7.03E-02 | 55.77814 | 45.0447 |
| | | F3 | 258.0353 | 9.65E-02 | 9.53E-03 | 16.06358 | 10.72216 |
| | D5 | F1 | 164.2846 | 6.34E-02 | 1.03E-02 | 12.81744 | 9.101874 |
| | | F2 | 944.685 | 0.157767 | 0.026019 | 30.73593 | 22.73455 |
| | | F3 | 53.16197 | 3.25E-02 | 1.25E-03 | 7.291274 | 4.80416 |
| | D6 | F1 | 2553.71 | 0.255362 | 5.56E-02 | 50.53457 | 39.71965 |
| | | F2 | 5198.402 | 0.381233 | -5.06E-02 | 72.10042 | 57.33464 |
| | | F3 | 2718.642 | 0.245573 | 4.38E-02 | 52.14094 | 39.49711 |
| 2nd size | D1 | F1 | 649.9882 | 6.24E-02 | 3.61E-02 | 25.49554 | 11.15004 |
| | | F2 | 1675.779 | 0.101312 | 5.13E-02 | 40.93736 | 18.68807 |
| | | F3 | 436.7934 | 4.79E-02 | 1.78E-02 | 20.90015 | 9.055494 |
| | D2 | F1 | 199.4485 | 6.43E-02 | 6.17E-03 | 14.123 | 9.730828 |
| | | F2 | 1104.712 | 0.15897 | 1.42E-02 | 33.23808 | 24.23059 |
| | | F3 | 84.39744 | 3.67E-02 | 1.48E-03 | 9.187051 | 5.632683 |
| | D3 | F1 | 451.3682 | 0.11066 | 1.33E-02 | 21.24599 | 15.5597 |
| | | F2 | 2168.024 | 0.255742 | 1.87E-02 | 46.56328 | 36.58924 |
| | | F3 | 185.1046 | 6.10E-02 | 3.66E-03 | 13.60567 | 8.715924 |
| | D4 | F1 | 627.9062 | 0.176486 | 3.13E-02 | 25.05872 | 18.97664 |
| | | F2 | 2760.522 | 0.371264 | -4.49E-02 | 52.54206 | 42.04366 |
| | | F3 | 212.9454 | 8.62E-02 | 8.23E-03 | 14.59303 | 9.541515 |
| | D5 | F1 | 167.0064 | 6.33E-02 | 1.03E-02 | 12.92344 | 9.424343 |
| | | F2 | 999.0452 | 0.159399 | 2.86E-02 | 31.60851 | 23.69769 |
| | | F3 | 49.41319 | 3.09E-02 | 1.08E-03 | 7.029638 | 4.783204 |
| | D6 | F1 | 2515.805 | 0.244773 | 6.16E-02 | 50.15912 | 39.22145 |
| | | F2 | 5124.662 | 0.361228 | -2.23E-02 | 71.58864 | 56.70345 |
| | | F3 | 2575.594 | 0.230928 | 4.99E-02 | 50.75165 | 38.31478 |
| 3rd size | D1 | F1 | 1.160481 | 2.40E-03 | 1.67E-05 | 1.077372 | 0.584638 |
| | | F2 | 8.382536 | 6.57E-03 | 1.83E-04 | 2.895572 | 1.599013 |
| | | F3 | 0.393049 | 7.94E-04 | 4.63E-06 | 0.627003 | 0.19395 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | D2 | F1 | 163.8329 | 5.93E-02 | 5.56E-03 | 12.80109 | 8.930487 |
| | | F2 | 927.0824 | 0.146365 | 9.72E-03 | 30.45129 | 22.32761 |
| | | F3 | 69.94551 | 3.46E-02 | 1.40E-03 | 8.36424 | 5.292427 |
| | D3 | F1 | 545.7631 | 0.126701 | 1.61E-02 | 23.36408 | 17.44561 |
| | | F2 | 2532.346 | 0.285071 | 1.48E-02 | 50.32782 | 40.16198 |
| | | F3 | 200.9987 | 6.66E-02 | 3.75E-03 | 14.17892 | 9.37331 |
| | D4 | F1 | 504.8545 | 0.143902 | 2.48E-02 | 22.47138 | 16.89616 |
| | | F2 | 2367.695 | 0.316867 | -4.50E-03 | 48.66419 | 38.83544 |
| | | F3 | 166.3961 | 7.06E-02 | 5.94E-03 | 12.90084 | 8.544518 |
| | D5 | F1 | 176.8065 | 5.88E-02 | 1.21E-02 | 13.29829 | 9.085389 |
| | | F2 | 958.7209 | 0.138563 | 2.77E-02 | 30.96654 | 21.67818 |
| | | F3 | 47.16735 | 2.78E-02 | 9.68E-04 | 6.868586 | 4.609955 |
| | D6 | F1 | 2161.564 | 0.210308 | 4.88E-02 | 46.49761 | 35.47179 |
| | | F2 | 4616.469 | 0.312364 | 4.82E-04 | 67.95189 | 52.37095 |
| | | F3 | 2091.233 | 0.192613 | 4.61E-02 | 45.7349 | 33.84188 |

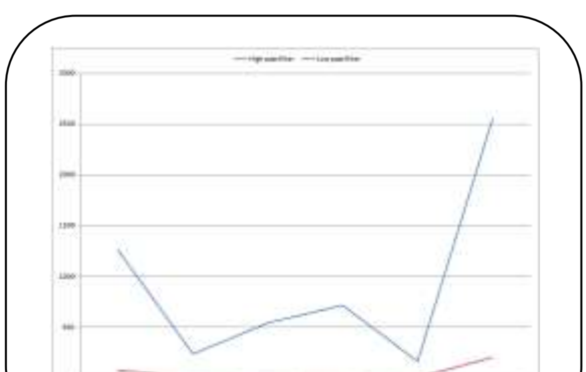
Table (2)
Represent experimental results for (Low pass filters)

| SIZE | IMAGE | Filter | M1 | M2 | M3 | M5 | M5 |
|---|---|---|---|---|---|---|---|
| 1st size | D1 | F1 | 74.56312 | 2.86E-02 | -3.14E-03 | 8.635051 | 5.663632 |
| | | F2 | 195.3313 | 4.74E-02 | -6.58E-03 | 13.97619 | 9.350586 |
| | | F3 | 122.3055 | 3.73E-02 | -4.62E-03 | 11.05925 | 7.377107 |
| | D2 | F1 | 7.347031 | 1.48E-02 | 7.36E-04 | 2.710559 | 1.887842 |
| | | F2 | 21.04507 | 2.59E-02 | -0.00102 | 4.58752 | 3.270642 |
| | | F3 | 12.94664 | 2.01E-02 | 1.39E-04 | 3.598168 | 2.552209 |
| | D3 | F1 | 16.76547 | 2.35E-02 | -5.57E-04 | 4.094592 | 2.945108 |
| | | F2 | 47.5927 | 0.040511 | -3.28E-03 | 6.898791 | 5.056662 |
| | | F3 | 29.23562 | 3.15E-02 | -1.59E-03 | 5.407033 | 3.945738 |
| | D4 | F1 | 23.19281 | 3.43E-02 | -5.11E-03 | 4.815923 | 3.610459 |
| | | F2 | 70.01915 | 0.061297 | -1.23E-02 | 8.367799 | 6.350704 |
| | | F3 | 42.29988 | 4.73E-02 | -8.33E-03 | 6.503878 | 4.926129 |
| | D5 | F1 | 5.269265 | 1.07E-02 | 9.77E-04 | 2.295503 | 1.578022 |
| | | F2 | 15.53992 | 1.91E-02 | -4.30E-04 | 3.942096 | 2.77781 |
| | | F3 | 9.502945 | 0.014761 | 5.42E-04 | 3.082705 | 2.160727 |
| | D6 | F1 | 202.4069 | 6.81E-02 | -1.48E-02 | 14.22707 | 10.63408 |
| | | F2 | 480.383 | 0.107033 | -2.62E-02 | 21.91779 | 16.55805 |
| | | F3 | 308.2096 | 0.085291 | -1.99E-02 | 17.55601 | 13.23687 |
| 2nd size | D1 | F1 | 37.91845 | 1.57E-02 | -1.51E-03 | 6.157959 | 3.086782 |
| | | F2 | 100.1122 | 2.61E-02 | -3.49E-03 | 10.00587 | 5.078484 |
| | | F3 | 62.67528 | 2.06E-02 | -2.36E-03 | 7.916981 | 4.018234 |
| | D2 | F1 | 6.480366 | 1.10E-02 | 1.66E-03 | 2.545724 | 1.686856 |
| | | F2 | 17.35986 | 1.89E-02 | 9.85E-04 | 4.166627 | 2.876381 |
| | | F3 | 10.94731 | 1.48E-02 | 1.53E-03 | 3.308759 | 2.258126 |
| | D3 | F1 | 13.31484 | 1.86E-02 | 5.57E-04 | 3.649046 | 2.643095 |
| | | F2 | 38.71466 | 3.25E-02 | -1.10E-03 | 6.222275 | 4.573014 |
| | | F3 | 23.62037 | 2.52E-02 | 2.10E-05 | 4.860208 | 3.561493 |
| | D4 | F1 | 20.49649 | 3.12E-02 | -3.85E-03 | 4.527424 | 3.316844 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | F2 | 60.65937 | 5.58E-02 | -1.00E-02 | 7.788619 | 5.853073 |
| | | F3 | 36.94535 | 4.31E-02 | -6.56E-03 | 6.078429 | 4.541356 |
| | D5 | F1 | 5.410179 | 0.010709 | 9.27E-04 | 2.326041 | 1.641351 |
| | | F2 | 15.92981 | 1.92E-02 | -4.70E-04 | 3.991323 | 2.887057 |
| | | F3 | 9.762063 | 1.48E-02 | 4.96E-04 | 3.124513 | 2.247344 |
| | D6 | F1 | 196.5977 | 6.48E-02 | -0.01319 | 14.0217 | 10.46187 |
| | | F2 | 471.9897 | 0.102627 | -2.35E-02 | 21.7259 | 16.43016 |
| | | F3 | 302.37 | 8.17E-02 | -1.78E-02 | 17.38925 | 13.1093 |
| 3rd size | D1 | F1 | 0.190946 | 7.78E-04 | 7.77E-04 | 0.437021 | 0.190517 |
| | | F2 | 0.231066 | 9.05E-04 | 7.77E-04 | 0.480745 | 0.221197 |
| | | F3 | 0.208324 | 8.41E-04 | 8.14E-04 | 0.456475 | 0.20575 |
| | D2 | F1 | 5.377387 | 1.03E-02 | 1.61E-03 | 2.319168 | 1.565329 |
| | | F2 | 14.22892 | 1.75E-02 | 9.62E-04 | 3.772529 | 2.644282 |
| | | F3 | 8.979833 | 1.37E-02 | 1.52E-03 | 2.996958 | 2.078309 |
| | D3 | F1 | 16.57992 | 2.16E-02 | -1.97E-04 | 4.072281 | 2.992705 |
| | | F2 | 48.10942 | 3.76E-02 | -2.49E-03 | 6.93684 | 5.175713 |
| | | F3 | 29.39305 | 2.92E-02 | -9.83E-04 | 5.422117 | 4.033255 |
| | D4 | F1 | 16.97189 | 2.50E-02 | -9.03E-04 | 4.120138 | 2.955589 |
| | | F2 | 47.61296 | 4.42E-02 | -4.71E-03 | 6.900955 | 5.154688 |
| | | F3 | 29.58764 | 3.43E-02 | -2.47E-03 | 5.440036 | 4.01888 |
| | D5 | F1 | 5.655653 | 1.01E-02 | 8.44E-05 | 2.378417 | 1.596009 |
| | | F2 | 17.18644 | 0.018148 | -1.68E-03 | 4.146098 | 2.808839 |
| | | F3 | 10.39713 | 0.013982 | -6.03E-04 | 3.224803 | 2.183437 |
| | D6 | F1 | 162.8856 | 5.45E-02 | -1.06E-02 | 12.76404 | 9.177001 |
| | | F2 | 386.1622 | 8.59E-02 | -1.82E-02 | 19.65312 | 14.357 |
| | | F3 | 249.2459 | 6.86E-02 | -1.40E-02 | 15.78922 | 11.48058 |

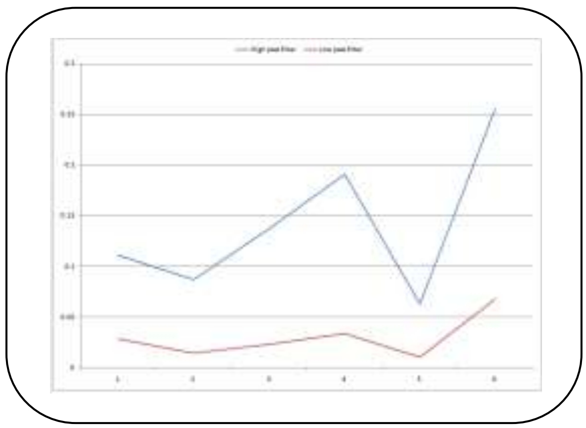
With
M1=MSE
M2=MAPE
M3=MPE
M4=RMSD
M5=MAE
And
F1= Enhancement Filter
F2= Sharpening Filter
F3= Smoothing Filter

From the above tables we can note that the out put images effected
With changing sizes and filters and the statistical measures effected and give different results in cases of changes in (texture size, texture kind and adoptive filter)

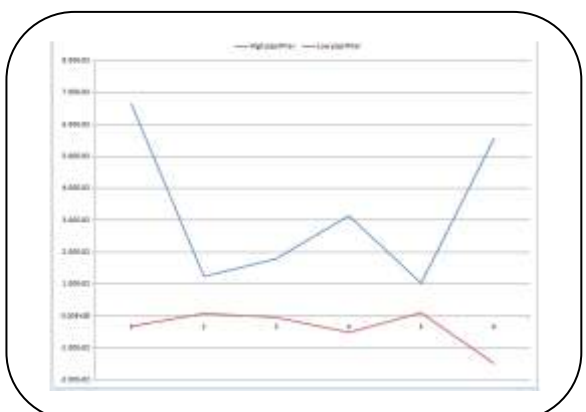


Fig(1)

Represent low and high pass filter according (MSE)



Represent low and high pass filter according (MAPE)



Fig(3)

Represent low and high pass filter according (MPE)

From the above figures we can note that output pixels for low pass filters with lowest changed comparing with output pixels for high pass filters and changing texture kind can give deferent result for all measures

## 12 CONCLUSION & SUGGESTION

From the above results we can give the following conclusion and suggestion
1-the out put pixels values affected with statically measure form
2- high pass filters give great canings comparing with low pass filters
3-enhanced, smooth and sharp filter effected with textural kind
4-output image changes depend on image size
5-measuring changes in input texture can be made by spectral, mathematical measures addition to statistical measures
6-taking deferent textures format (bmp, gif ,..., etc) can give deferent output pixel values

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